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SHEWHART $\bar{X}$ AND R CONTROL CHARTS USING NEW RANKED SET SAMPLING SCHEMES

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Abstract: In this study, we proposed Shewhart $\bar{X}$ and R control charts using some ranked-based sampling designs when the parent distribution follows a bivariate normal distribution. The performance of the mean and range charts based on simple random sampling and the ranked-based designs are compared using Monte Carlo simulation. Simulation results show that our proposed control charts give more efficient estimates compared to classical simple random sampling scheme.

Key words: Control charts; Ranked set sampling; Median ranked set sampling; Neoteric ranked set sampling.

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1. Introduction

Control charts are graphical techniques for continuous monitoring of the quality of a manufacturing process [5]. The basic structure of a control charts consist of upper and lower control limits and the natural variations are expected to be within these limits. There are different kinds of control charts such as mean, range, EWMA etc. Taking into account the sampling schemes in control charts is an important issue. In literature, Pongpullponsak and Sontisamran [6] defined Shewhart control charts on ranked set sampling (RSS), Yaqub et al. [8] used modified succesive sampling for mean charts. Abujiya and Lee [2] examined three control charts using ranked set sampling.

In sampling literature, some authors tried to modify ranked set sampling to obtain more efficient statistical inference. Recently, Zamanzade and Al-Omari [9] have introduced neoteric ranked sampling (NRSS) and showed that the mean estimator in NRSS design is more efficient than its competitor in ranked set sampling and simple random sampling (SRS). Koyuncu [4] studied median ranked set sampling (MRSS) and showed its superiority over existing methods. In this paper following Koyuncu [4], Zamanzade and Al-Omari [9], we have proposed $\bar{X}$ and R charts under different ranked set sampling schemes.

2. Estimations

2.1. The Location Estimation

The location estimator that we consider is the mean of the sample means,

$$\bar{\bar{X}} = \frac{1}{k}\sum_{i=1}^{k}\bar{X}_i = \frac{1}{k}\sum_{i=1}^{k}(\frac{1}{n}\sum_{j=1}^{n}X_{ij}) \tag{2.1}$$

where $i = 1, 2, ..., k$ and $j = 1, 2, ..., n$.

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2.2. The Scale Estimation

We consider the mean of sample range as scale estimator given by

$$\bar{R} = \frac{1}{k}\sum_{i=1}^{k} R_i \tag{2.2}$$

where R_i: the range of the ith sample.

3. Sampling Methods

In this section we give some notations of different sampling schemes which are used to construct control charts.

3.1. Simple Random Sampling Design

Let $(X_1,Y_1),(X_2,Y_2),\ldots,(X_n,Y_n)$ be a bivariate random sample from a population of size N with pdf $f(x,y)$, means μ_x, μ_y, variances σ_x^2, σ_y^2 and correlation coefficient ρ_{xy}. Assume that the auxiliary variable X is correlated with the variable of interest Y. The unbiased estimators of the population mean of study and auxiliary variables based on SRS are as follows: $\bar{Y}_{(SRS)} = \frac{1}{n}\sum\limits_{i=1}^{n} Y_i$; $\bar{X}_{(SRS)} = \frac{1}{n}\sum\limits_{i=1}^{n} X_i$. The range of sample for study and auxiliary variable can be calculated using following equations respectively:

$$R_{Y(SRS)} = max(Y_i) - min(Y_i); \tag{3.1}$$
$$R_{X(SRS)} = max(X_i) - min(X_i) \tag{3.2}$$

where $i = 1,2,\ldots,n$

3.2. Ranked Set Sampling Design

RSS design can be described as follows:

1. Select a simple random sample of size n^2 units from the target finite population and divide them into n samples each of size n.
2. Rank the units within each sample in increasing magnitude by using personal judgment, eye inspection or based on a concomitant variable.
3. Select the ith ranked unit from the ith sample
4. Repeat steps 1 through 3, m times if needed to obtain a RSS of size $N = nm$.

Let

$$\begin{array}{c}
(X_{11h},Y_{11h})\ ,\ (X_{12h},Y_{12h}),\ldots,(X_{1nh},Y_{1nh});\\
(X_{21h},Y_{21h})\ ,\ (X_{22h},Y_{22h}),\ldots,(X_{2nh},Y_{2nh});\\
\vdots\ ;\\
(X_{n1h},Y_{n1h})\ ,\ (X_{n2h},Y_{n2h}),\ldots,(X_{nnh},Y_{nnh})
\end{array}$$

be n independent bivariate random samples with pdf $f(x,y)$, each of size n in the hth cycle, $(h = 1,2,\ldots,m)$. Let

$$(X_{i(1:n)h},Y_{i[1:n]h}),(X_{i(2:n)h},Y_{i[2:n]h}),\ldots,(X_{i(n:n)h},Y_{i[n:n]h})$$

be the order statistics of $X_{i1h},X_{i2h},\ldots,X_{inh}$ and the judgement order of $Y_{i1h},Y_{i2h},\ldots,Y_{inh}$ $(i = 1,2,\ldots,n)$, where () and [] indicate that the ranking of X is perfect and ranking of Y has errors. Assume measured units using RSS are

$$(X_{1(1:n)h},Y_{1[1:n]h}),(X_{2(2:n)h},Y_{2[2:n]h}),\ldots,(X_{n(n:n)h},Y_{n[n:n]h}).$$

Then the RSS estimators of population mean for study and auxiliary variable can be written as

$$\bar{Y}_{(RSS)} = \frac{1}{nm}\sum_{h=1}^{m}\sum_{i=1}^{n} Y_{i[i:n]h}; \tag{3.3}$$

$$\bar{X}_{(RSS)} = \frac{1}{nm}\sum_{h=1}^{m}\sum_{i=1}^{n} X_{i(i:n)h}. \tag{3.4}$$

The RSS estimators of population range for study and auxiliary variable can be written as

$$R_{Y(RSS)} = max(Y_{i[i:n]h}) - min(Y_{i[i:n]h}); \tag{3.5}$$

$$R_{X(RSS)} = max(X_{i(i:n)h}) - min(X_{i(i:n)h}) \tag{3.6}$$

where $(i = 1, 2, \ldots, n)$.

3.3. Median Ranked Set Sampling Design

For the sake of brevity we follow Al-Omari [1]' sampling design and notations. MRSS design can be described as in the following steps:

1. Select n random samples each of size n bivariate units from the population of interest.
2. The units within each sample are ranked by visual inspection or any other cost free method with respect to a variable of interest.
3. If n is odd, select the $((n+1)/2)$ th-smallest ranked unit X together with the associated Y from each set, i.e., the median of each set. If n is even, from the first $n/2$ sets select the $(n/2)$th ranked unit X together with the associated Y and from the other sets select the $((n+2)/2)$ th ranked unit X together with the associated Y .
4. The whole process can be repeated m times if needed to obtain a sample of size nm units.

Let $(X_{i(1)h}, Y_{i[1]h}), (X_{i(2)h}, Y_{i[2]h}), \ldots, (X_{i(n)h}, Y_{i[n]h})$ be the order statistics of $X_{i1h}, X_{i2h}, \ldots, X_{inh}$ and the judgement order of $Y_{i1h}, Y_{i2h}, \ldots, Y_{inh}$ $(i = 1, 2, \ldots, n), (h = 1, 2, \ldots, m)$ where () and [] indicate that the ranking of X is perfect and ranking of Y has errors. For odd and even sample sizes the units measured using MRSS are denoted by MRSSO and MRSSE, respectively. For odd sample size let

$$(X_{1(\frac{n+1}{2})h}, Y_{1[\frac{n+1}{2}]h}), (X_{2(\frac{n+1}{2})h}, Y_{2[\frac{n+1}{2}]h}), \ldots, (X_{n(\frac{n+1}{2})h}, Y_{n[\frac{n+1}{2}]h})$$

denote the observed units by MRSSO. The sample mean of X and Y are given as following respectively,

$$\bar{Y}_{(MRSSO)} = \frac{1}{nm}\sum_{h=1}^{m}\sum_{i=1}^{n} Y_{i[\frac{n+1}{2}]h}; \tag{3.7}$$

$$\bar{X}_{(MRSSO)} = \frac{1}{nm}\sum_{h=1}^{m}\sum_{i=1}^{n} X_{i(\frac{n+1}{2})h}. \tag{3.8}$$

For even sample size let $(X_{1(\frac{n}{2})h}, Y_{1[\frac{n}{2}]h}), (X_{2(\frac{n}{2})h}, Y_{2[\frac{n}{2}]h}), \ldots, (X_{\frac{n}{2}(\frac{n}{2})h}, Y_{\frac{n}{2}[\frac{n}{2}]h})$, $(X_{\frac{n+2}{2}(\frac{n+2}{2})h}, Y_{\frac{n+2}{2}[\frac{n+2}{2}]h}), (X_{\frac{n+4}{2}(\frac{n+2}{2})h}, Y_{\frac{n+4}{2}[\frac{n+2}{2}]h}), \ldots, (X_{n(\frac{n}{2})h}, Y_{n[\frac{n}{2}]h})$ denote the observed units by MRSSE. The sample mean of X and Y are given respectively

$$\bar{Y}_{(MRSSE)} = \frac{1}{nm}\sum_{h=1}^{m}(\sum_{i=1}^{\frac{n}{2}} Y_{i[\frac{n}{2}]h} + \sum_{i=\frac{n+2}{2}}^{n} Y_{i[\frac{n+2}{2}]h}) \tag{3.9}$$

$$\bar{X}_{(MRSSE)} = \frac{1}{nm}\sum_{h=1}^{m}(\sum_{i=1}^{\frac{n}{2}} X_{i(\frac{n}{2})h} + \sum_{i=\frac{n+2}{2}}^{n} X_{i(\frac{n+2}{2})h}). \tag{3.10}$$

The ranges using MRSS can be calculated if the sample size odd

$$R_{Y(MRSSO)} = max(Y_{i[\frac{n+1}{2}]h}) - min(Y_{i[\frac{n+1}{2}]h}); \tag{3.11}$$
$$R_{X(MRSSO)} = max(X_{i(\frac{n+1}{2})h}) - min(X_{i(\frac{n+1}{2})h}) \tag{3.12}$$

for $(i = 1, 2, \ldots, n)$; if the sample size even

$$R_{Y(MRSSE)} = max(Y_{i[\frac{n}{2}]h}, Y_{i[\frac{n+2}{2}]h}) - min(Y_{i[\frac{n}{2}]h}, Y_{i[\frac{n+2}{2}]h}); \tag{3.13}$$
$$R_{X(MRSSE)} = max(X_{i(\frac{n}{2})h}, X_{i(\frac{n+2}{2})h}) - min(X_{i(\frac{n}{2})h}, X_{i(\frac{n+2}{2})h}). \tag{3.14}$$

3.4. Neoteric Ranked Set Sampling Design

Zamanzade and Al-Omari [9] have defined a new neoteric ranked set sampling. The NRSS scheme can be described as follows:

1. Select a simple random sample of size n^2 units from the target finite population.

2. Ranked the n^2 selected units in an increasing magnitude based on a concomitant variable, personel judgment or any inexpensive method.

3. If n is an odd, then select the $[\frac{n+1}{2} + (i-1)n]$ th ranked unit for $(i = 1, 2, \ldots, n)$. If n is an even, then select the $[l + (i-1)n]$ th ranked unit, where $[l = \frac{n}{2}]$ if i is an even and $[l = \frac{n+2}{2}]$ if i is an odd for $(i = 1, 2, \ldots, n)$.

4. Repeat steps 1 through 3 m times if needed to obtain a NRSS of size $N = nm$.

Let $(X_{1h}, Y_{1h}), (X_{2h}, Y_{2h}), \ldots, (X_{n^2h}, Y_{n^2h})$ be n^2 simple random units selected from the population with the pdf $f(x,y)$ and let $(X_{(1)h}, Y_{[1]h}), (X_{(2)h}, Y_{[2]h}), \ldots, (X_{(n^2)h}, Y_{[n^2]h})$ be order statistics of measured units by NRSS $(h = 1, 2, \ldots, m)$. The sample means of study and auxiliary variables under NRSS scheme are given as following

$$\bar{Y}_{(NRSS)} = \frac{1}{nm}\sum_{h=1}^{m}\sum_{i=1}^{n} Y_{[(i-1)n+l]h}; \tag{3.15}$$
$$\bar{X}_{(NRSS)} = \frac{1}{nm}\sum_{h=1}^{m}\sum_{i=1}^{n} X_{[(i-1)n+l]h}. \tag{3.16}$$

The the ranges of study and auxiliary variables under NRSS scheme are given as following

$$R_{Y(NRSS)} = max(Y_{[i]h}) - min(Y_{[i]h}); \tag{3.17}$$
$$R_{X(NRSS)} = max(X_{(i)h}) - min(X_{(i)h}). \tag{3.18}$$

4. Control Chart Methods

4.1. Shewhart $\bar{X}$ Chart

We first consider the Shewhart method proposed by Montgomery [5]. The control limits of mean chart for Shewhart method are given as follows:

$$UCL_{Shewhart} = \bar{\bar{Y}} + \frac{3}{d_2\sqrt{n}}\bar{R}_Y \tag{4.1}$$
$$LCL_{Shewhart} = \bar{\bar{Y}} - \frac{3}{d_2\sqrt{n}}\bar{R}_Y. \tag{4.2}$$

where $\bar{\bar{Y}}$ is the mean of the subgroups means and d_2 is constant that depends on the subgroup size n, and is calculated when the distribution is normal [5].

4.2. Shewhart $\bar{X}$ Charts Limits under Different Ranked Set Sampling

The control limits of the mean chart for Shewhart method using different sampling designs are defined as follows:

$$
\begin{aligned}
UCL_{Shewhart(j)} &= \bar{\bar{Y}}_{(j)} + \frac{3}{d_2\sqrt{n}}\bar{R}_{Y(j)} \\
LCL_{Shewhart(j)} &= \bar{\bar{Y}}_{(j)} - \frac{3}{d_2\sqrt{n}}\bar{R}_{Y(j)}.
\end{aligned} \tag{4.3}
$$

where j implies the sampling methods which are SRS, RSS, MRSS and NRSS sampling designs.

$\bar{\bar{Y}}_j$ can be rewriten

$$
\begin{aligned}
\bar{\bar{Y}}_{(SRS)} &= \frac{1}{k}\sum_{i=1}^{k}\bar{Y}_{(SRS)}, \quad \bar{\bar{Y}}_{(RSS)} = \frac{1}{k}\sum_{i=1}^{k}\bar{Y}_{(RSS)} \\
\bar{\bar{Y}}_{(MRSS)} &= \frac{1}{k}\sum_{i=1}^{k}\bar{Y}_{(MRSS)}, \quad \bar{\bar{Y}}_{(NRSS)} = \frac{1}{k}\sum_{i=1}^{k}\bar{Y}_{(NRSS)}
\end{aligned} \tag{4.4}
$$

and $\bar{R}_j$ is

$$
\begin{aligned}
\bar{R}_{Y(SRS)} &= \frac{1}{k}\sum_{i=1}^{k}R_{Y(SRS)}, \quad \bar{R}_{Y(RSS)} = \frac{1}{k}\sum_{i=1}^{k}R_{Y(RSS)}, \\
\bar{R}_{Y(MRSS)} &= \frac{1}{k}\sum_{i=1}^{k}R_{Y(MRSS)}, \quad \bar{R}_{Y(NRSS)} = \frac{1}{k}\sum_{i=1}^{k}R_{Y(NRSS)}
\end{aligned} \tag{4.5}
$$

where k is the number of samples.

4.3. Shewhart R Charts

The conventional range control chart when the distribution is normal is the Shewhart R chart. We first consider the Shewhart method proposed by [5]. The control limits of R chart for Shewhart method are given as follows:

$$
\begin{aligned}
UCL_{R_{Shewhart}} &= (1 + \frac{3d_3}{d_2})\bar{R}, \\
LCL_{R_{Shewhart}} &= (1 - \frac{3d_3}{d_2})\bar{R}.
\end{aligned}
$$

where d_2 and d_3 are constants that depend on the subgroup size n, and are calculated when the distribution is normal [5].

4.4. Shewhart R Charts Limits under Different Ranked Set Sampling

The control limits of the R chart for Shewhart method using different ranked set sampling schemes are defined as follows:

$$
\begin{aligned}
UCL_{R_{Shewhartj}} &= (1 + \frac{3d_3}{d_2})\bar{R}_j, \\
LCL_{R_{Shewhartj}} &= (1 - \frac{3d_3}{d_2})\bar{R}_j.
\end{aligned} \tag{4.6}
$$

where j implies the ranked sample methods and $\bar{R}_j$ is

$$
\begin{aligned}
\bar{R}_{Y(SRS)} &= \frac{1}{k}\sum_{i=1}^{k}R_{Y(SRS)}, \quad \bar{R}_{Y(RSS)} = \frac{1}{k}\sum_{i=1}^{k}R_{Y(RSS)}, \\
\bar{R}_{Y(MRSS)} &= \frac{1}{k}\sum_{i=1}^{k}R_{Y(MRSS)}, \quad \bar{R}_{Y(NRSS)} = \frac{1}{k}\sum_{i=1}^{k}R_{Y(NRSS)}
\end{aligned} \tag{4.7}
$$

where k is the number of samples.

4.5. Determination of the Control Charts Constants

The Shewart method constants d_2 and d_3 are calculated by taking the mean and standard deviation of relative range $\left(\frac{R}{\sigma}\right)$, respectively. All constants are obtained for normal distributions via simulation. We obtain $E(\bar{R})$ by simulation: we generate 100.000 times k samples of size n, compute R for each instance and take the average of the values. The results for all constants for $k = 30$ are presented in Table 1 for $n = 3, 5, 7, 10$.

TABLE 1. The constants of Shewart $\bar{X}$ and R charts for different sample sizes

Constant/ n	3	5	7	10
d2	1.6926	2.3260	2.7044	3.0775
d3	0.8883	0.8641	0.8332	0.7971

4.6. Performance of Shewhart Charts based on Different Sampling Desings

In the process control, if we do not know the the parameters of the process, we need to estimate the unknown parameters. Therefore control charts can be applied in a two-phase procedure. In Phase I, control charts are used to define the in-control state of the process. The parameters of the process are estimated from Phase I sample and control limits are estimated for using in Phase II. In Phase II, samples from the process are prospectively monitored for departures from the in-control state. The Type I risk indicates the probability of a subgroup $\bar{X}$ and R falling outside the ± 3 sigma control limits. When the process is in-control, the Type I risks are 0.27%. However, due to the control limits, about 0.0027 of all control points will be false alarms and have no assignable cause of variation. The ARL is the number of points plotted within the control limits before one exceeds the limits. Under the normality assumption and for the Shewhart control charts, it is expected that 370.4 points would be plotted on the chart within the 3σ control limits, before one gets out.

A comparison between the performances of the Shewhart $\bar{X}$ and R control charts for monitoring the process is made in terms of the Type I risk probabilities and the average run length values.

Let E_i denote the event that the $i-th$ sample mean is beyond the limits. Further, denote by $P(Ei|\bar{\bar{X}}, \hat{\sigma})$ the conditional probability that for given $\bar{\bar{X}}$ and $\hat{\sigma}$, the sample mean $\bar{X}_i$ is beyond the control limits

$$P(Ei|\bar{\bar{X}}, \hat{\sigma}) = P(\bar{X}_i < LCL \quad or \quad \bar{X}_i > UCL) \tag{4.8}$$

Given $\bar{\bar{X}}$ and $\hat{\sigma}$, the events E_s and E_t $(s \neq t)$ are independent. Therefore, the run length has a geometric distribution with parameter $P(Ei|\bar{\bar{X}}, \hat{\sigma})$. When we take the expectation over the estimation data X_{ij} we get the probability of one sample showing a Type I false alarm for $\bar{X}$ chart

$$P(Ei|\bar{\bar{X}}, \hat{\sigma}) = E(P(Ei|\bar{\bar{X}}, \hat{\sigma})) \tag{4.9}$$

and, similarly, the average run length (ARL)

$$ARL = E(1/P(Ei|\bar{\bar{X}}, \hat{\sigma})). \tag{4.10}$$

We get the Type I false alarm for R chart like as $\bar{X}$ chart.

These expectations are simulated by generating 10 000 times k data samples of size n, computing for each data set the conditional value and averaging the conditional values over the data sets. Note that for the calculation of the control limits in Phase I the process is considered to be in-control [7].

5. Simulation Study

In this section, we conducted a simulation study to compare the efficiency of proposed control charts with existing charts using R programme version Ri386 3.2.3 setting seed=200. We consider Shewhart $\bar{X}$ and R control charts under different sampling schemes for normal distributed data. We use the mean and the range estimators of the standard deviation for the Shewhart method. We obtain p and the in-control ARL for moderate sample size (30 subgroups of 3-10) for bivariate normal distribution.

Simulation consists of two segments. The steps of each segment are described as below:

Phase 1:

1.a. Generate correlated skewed finite population using Bivariate Normal Distribution size N=1000.

1.b. Select samples size n from Normal distribution ($\mu = 2, \sigma^2 = 10$) varieties for $n = 3, 5, 7, 10$ using different sampling schemes with SRS, RSS, MRSS and NRSS respectively.

1.c. Repeat step 1.b 30 times ($k = 30$) .

1.d. Compute the control limits using the Equations (4.3) and (4.6) for Shewart $\bar{X}$ and R charts, respectively.

Phase 2:

2.a. Generate correlated finite population using Bivarite Normal Distribution as described in step 1.a.

2.b. Select samples size n from Normal distribution ($\mu = 2, \sigma^2 = 10$) varieties for $n = 3, 5, 7, 10$ using different sampling schemes with SRS, RSS, MRSS and NRSS respectively, as in step 1.b.

2.c. Repeat step 2.b 100 times ($k = 100$).

2.d. Compute the sample statistics for three mean charts for different sampling schemes.

2.e. Record whether the sample statistics calculated in step 2.d are within the control limits of step 1.d. or not for all methods.

2.f. Repeat steps 1.a through 2.d, 10000 times and obtain an average Type I risk for each method.

We present the results of the simulation for n=3,5,7,10. Table 2 and Table 3 give the results of p and ARL for the $\bar{X}$ and R control chart by using ranked set sampling schemes for bivariate normal distributed data, respectively.

6. Results

In the statistical process control, the desired ARL value of 370 indicates that the control limits are chosen to provide p of 0.0027. If p is to be as low as possible and ARL is to be as high as possible, it means that the process in control.

To compare these proposed range charts by using different ranked set sampling designs for $n = 3, 5, 7, 10$, the simulation run in the above Section 5. The results of the p and ARL values for the $\bar{X}$ and R charts by using ranked set sampling schemes for bivariate normal distribution are presented in Tables 2 and 3. The most important points of this study can be sum up as following:

• For the symmetric distributions, Shewhart mean charts under MRSS and SRS give the desirable results.

• For the symmetric distributions, the Neoteric and RSS Shewhart range charts give the desirable results. The Neoteric range chart has the best performance. It is important to see that The Neoteric and RSS Shewhart can be used under normality.

Table 2. The results of p for the Shewart $\bar{X}$ and R charts based on different sample sizes and sampling designs

	$\bar{X}$ chart			
n	**SRS**	**RSS**	**NRSS**	**MRSS**
3	0.004555	0.000871	0.000813	0.004693
5	0.003862	0.000468	0.000370	0.004253
7	0.003805	0.000337	0.000257	0.004288
10	0.003679	0.000271	0.000203	0.005382
	R chart			
n	**SRS**	**RSS**	**NRSS**	**MRSS**
3	0.008922	0.006985	0.005312	0.008739
5	0.006279	0.005058	0.003441	0.006552
7	0.005888	0.004443	0.003091	0.006223
10	0.005730	0.004671	0.002858	0.006029

Table 3. The results of ARL for the Shewart $\bar{X}$ and R charts based on different sample sizes and sampling designs

	$\bar{X}$ chart			
n	**SRS**	**RSS**	**NRSS**	**MRSS**
3	219.5390	1148.106	1230.012	213.0833
5	258.9332	2136.752	2702.703	235.1281
7	262.8121	2967.359	3891.051	233.2090
10	271.8130	3690.037	4926.108	185.8045
	R chart			
n	**SRS**	**RSS**	**NRSS**	**MRSS**
3	112.0825	143.1639	188.2530	114.4296
5	159.2610	197.7066	290.6132	152.6252
7	169.8370	225.0731	323.5199	160.6942
10	174.5201	214.0869	349.8950	165.8650

7. Conclusion

In many practical situations, the ranking process is done using a concomitant variable. In some case, the actual quantifications of the concomitant variable along with the ith ordered value of the variable of interest are available. In those cases, an improved statistical inference is expected when the measured values of concomitant variables are incorporated into statistical inference. This idea was firstly proposed by [3] for estimation of the population mean and pursued later by [11] and [10] for the population variance and proportion, respectively. In this paper, we propose to use new ranked set sampling schemes to construct the limits of $\bar{X}$ and R charts based on Shewhart method. The considered sampling designs are the simple random sampling, ranked set sampling, median rank set sampling and neoteric ranked set sampling. To evaluate the $\bar{X}$ and R charts performance for bivariate normal distributed data we obtain the p and ARL values by using Monte Carlo simulation. In simulation we consider small and large sample sizes and different sampling schemes. The results can be end up as follows: The Neoteric and RSS Shewhart R charts can be used in the case of normality, the Neoteric range chart has the best performance. Shewhart $\bar{X}$ charts under MRSS and SRS have good performance.

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