İSTATİSTİK: JOURNAL OF THE TURKISH STATISTICAL ASSOCIATION
Vol. 7, No. 3, December 2014, pp. 87–98
ISSN 1300-4077 | 14 | 3 | 87 | 98

İSTATİSTİK

# A PROFICIENT RANDOMIZED RESPONSE MODEL

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***Abstract:*** In this article, we have suggested a new randomized response model and its properties have been studied. The proposed model is found to be more efficient than the randomized response models studied by Bar – Lev et al. (2004) and Eichhorn and Hayre (1983). The relative efficiency of the proposed model has been studied with respect to the Bar – Lev et al.'s (2004) and Eichhorn and Hayre's (1983) models. Numerical illustrations are also given in support of the present study.

*Key words*: Randomized response sampling,Estimation of proportion, Respondents protection, sensitive quantitative variable
*History*: Submitted: 14 January 2015; Revised: 30 March 2015; Accepted: 7 April 2015

## 1. Introduction

Randomized response technique (RRTs) have been extensively used for personal interview surveys ever since the pioneering work of Warner (1965). The main aim of such procedures and techniques is to estimate the proportion of a population whose truthful response to a sensitive question would be "Yes" without exposing the respondents to the interviewer, and consequently avoiding social stigma or fear reprisals. Several randomized response models have been developed by researchers for collecting data on both the qualitative and the quantitative variables. For details, one can refer to Fox and Tracy (1986), Grewal et al. (2005-2006), Hong (2005-2006), Ryu et al. (2005-2006), Mahajan et al. (2007), Perri (2008), Singh and Chen (2009), Odumade and Singh (2009, 2010), Singh and Tarray (2012, 2013, 2014) and Barabesi et al. (2014) etc. Eichorn and Hayre (1983) suggested a multiplicative model to collect information on sensitive quantitative variables like income, tax evasion, amount of drug used etc. Let $X$ be the true response and S be some scrambling variable, independent of $X$, with known mean $\theta$ and standard deviation $\sigma_s$ . The respondent is asked to report the response $Z$ as given by

$$Z = \frac{SX}{\theta} \tag{1.1}$$

Since $E(Z) = E(X) = \mu_x$ . For estimating the population mean $\mu_x$ , a sample of size n is taken using simple random sample with replacement (SRSWR): Then an unbiased estimator of the population mean $\mu_x$ of $X$ is given by

$$\hat{\mu}_{x(EH)} = \overline{Z} = \frac{1}{n}\sum_{i=1}^{n} Z_i \tag{1.2}$$

The variance of $\hat{\mu}_{x(EH)}$ is given by

$$V\left(\hat{\mu}_{x(EH)}\right) = \frac{\mu_x^2}{n}\left[C_x^2 + C_\gamma^2(1 + C_x^2)\right] \tag{1.3}$$

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where $C_\gamma = \sigma_s/\theta$, $C_x = (\sqrt{V(x)}/\mu_x) = \sigma_x/\mu_x$ are the coefficient of variation of scrambling variable $S$ and the study variable $X$.

We note that $P(S=1)=1$, then the Eichhorn and Hayre (1983) technique out to be a direct interview, a fact which exposes the interviewee's response to the sensitive question. According, Eichhorn and Hayre (1983) have discussed different choices of $S$ for which $P(S=1)=0$ as well as various alternatives for the distribution of $S$ so as to make the variance of $\hat{\mu}_{x(EH)}$ as small as possible.

We shall now discuss a randomized response model envisaged by Bar – Lev, Bobovitch, and Boukai (2004; the BBB model). In the BBB model, the distribution of the responses is given by:

$$Z_i = \begin{cases} X_i S & \text{with probability } (1-P) \\ X_i & \text{with probability } P \end{cases} \tag{1.4}$$

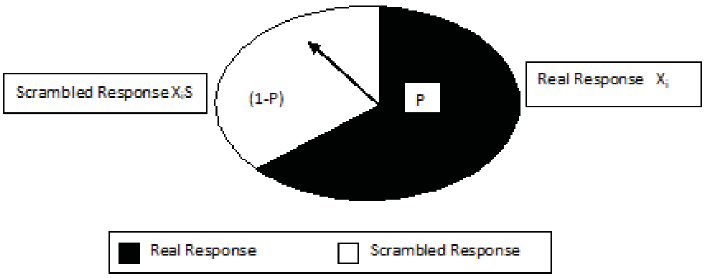


FIGURE 1. Bar – lev, Bobovitch and Boukai (2004; BBB) randomized response device

In other words, each respondent is requested to rotate a spinner unobserved by the interviewer, and if the spinner stops in the shaded area, then the respondent is request to report the real response on the sensitive variable, say $X_i$; and if the spinner stops in the non shaded area, then the respondent is required to report the scrambled response, say $X_iS$, where $S$ is the scrambled variable. Let $P$ be the radial non shaded area of the spinner as shown in Fig.1

An unbiased estimator of the population mean $\mu_x$ is given by:

$$\hat{\mu}_{x(BBB)} = \frac{\sum_{i=1}^{n} Z_i}{n\{(1-P)\theta + P\}} \tag{1.5}$$

with variance under SRSWR sampling given by

$$\hat{\mu}_{x(BBB)} = \frac{\mu_x^2}{n}\left[C_x^2 + (1+C_x^2)C_P^2\right], \tag{1.6}$$

where

$$C_P^2 = \left\{\frac{(1-P)\theta^2(1+C_\gamma^2)+P}{\{(1-P)\theta+P\}^2} - 1\right\} \tag{1.7}$$

In the next section, we suggest a new randomized response whose description is as below

## 2. Proposed Randomized Response Model

We note that the procedure due to Bar – Lev et al. (2004) uses a known design parameter $P$ $(0 < P < 1)$ controlled by the experimenter. As the design parameter $P$ is known, therefore it is logical to request respondents to use the knowledge of $P$ while giving the response as also suggested by Odumade and Singh (2009, 2010). We assume that a sample of size $n$ is selected by simple random sample with replacement (SRSWR). In the proposed procedure, interviewee's response to the sensitive question is:

$$Z_i = \begin{cases} \frac{X_i S}{(1-P)\theta} & \text{with probability } (1-P) \\ \frac{X_i}{P} & \text{with probability } P \end{cases} \tag{2.1}$$

The expected value of $Z_i$ is given by

$$\begin{aligned} E(Z_i) &= (1-P)\frac{E(X_i)E(S)}{(1-P)\theta} + P\frac{E(X_i)}{P} \\ &= \frac{E(X_i)(1-P)\theta}{(1-P)\theta} + P\frac{E(X_i)}{P} \\ &= 2E(X_i) = 2\mu_x \end{aligned}$$

Thus an unbiased estimator of $\mu_x$ is given by

$$\hat{\mu}_{x(ST1)} = \frac{\overline{Z}}{2} = \frac{1}{n}\sum_{i=1}^{n}\frac{Z_i}{2} \tag{2.2}$$

The variance $\hat{\mu}_{x(ST1)}$is given by

$$\frac{V(Z_i)}{4n} \tag{2.3}$$

The variance of $Z_i$ is observed as follows:

$$\begin{aligned} V(Z_i) &= [E(Z_i^2) - (E(Z_i))^2] \\ &= E(Z_i^2) - \mu_x^2 \\ &= (1-P)\frac{E(X_i^2)E(S^2)}{\theta^2(1-P)^2} + \frac{PE(X_i^2)}{p^2} - \mu_x^2 \\ &= E(X_i^2)\left[\frac{\theta^2(1+C_\gamma^2)}{\theta^2(1-P)} + \frac{1}{P}\right] - \mu_x^2 \\ &= E(X_i^2)\left[\frac{P(1+C_\gamma^2)+(1-P)}{P(1-P)}\right] - \mu_x^2 \\ &= \frac{\mu_x^2(1+C_x^2)(1+PC_\gamma^2)}{P(1-P)} - \mu_x^2 \\ &= \mu_x^2\left[\frac{(1+C_x^2)(1+PC_\gamma^2)}{P(1-P)} - 1\right] \end{aligned} \tag{2.4}$$

Thus the variance of $\hat{\mu}_{x(ST1)}$ is given by

$$\begin{aligned} V(\hat{\mu}_{x(ST1)}) &= \frac{\mu_x^2}{4n}\left[\frac{(1+C_x^2)(1+PC_\gamma^2)}{P(1-P)} - 1\right] \\ &= \frac{\mu_x^2}{4n}\left[C_x^2 + C_{P1}^2(1+C_x^2)\right] \end{aligned} \tag{2.5}$$

where

$$C_{P1}^2 = \left[\frac{(1+PC_\gamma^2)}{P(1-P)} - 1\right]$$

.

From (1.3) and (2.5) we have

$$V(\hat{\mu}_{x(EH)}) - V(\hat{\mu}_{x(ST1)}) = \frac{\mu_x^2}{4n}\left[1+4C_x^2+(1+C_x^2)\left\{4C_\gamma^2 - \frac{(1+PC_\gamma^2)}{P(1-P)}\right\}\right]$$

which is always positive if

$$\left[3C_x^2+(1+C_x^2)\left\{1+4C_\gamma^2 - \frac{(1+PC_\gamma^2)}{P(1-P)}\right\}\right] > 0 \quad (2.6)$$

Thus the proposed estimator $\hat{\mu}_{x(ST1)}$ is more efficient than Eichhorn and Hayre's (1983) estimator $\hat{\mu}_{x(EH)}$ as long as condition (2.6) is satisfied.

From (1.6) and (2.5) we have

$$V(\hat{\mu}_{x(BBB)}) - V(\hat{\mu}_{x(ST1)}) = \frac{\mu_x^2(1+C_x^2)(C_P^2 - C_{P1}^2)}{P(1-P)}$$

which is positive if

$$(C_P^2 - C_{P1}^2) > 0$$

i.e if

$$\frac{\left\{(1-P)\theta^2(1+C_\gamma^2)+P\right\}}{\left\{(1-P)\theta+P\right\}^2} > \frac{\left\{1+PC_\gamma^2\right\}}{P(1-P)} \quad (2.7)$$

Thus the proposed estimator $\hat{\mu}_{x(ST1)}$ is more efficient than the Bar – Lev et al. (2004) estimator $\hat{\mu}_{x(BBB)}$ if the condition (2.7) is satisfied.

To see the merits of the proposed unbiased estimator $\hat{\mu}_{x(ST1)}$ we have computed the percent relative efficiency (PRE) of $\hat{\mu}_{x(ST1)}$ with respect to the estimators $\hat{\mu}_{x(EH)}$ and $\hat{\mu}_{x(BBB)}$ by using the formulae:

$$PRE(\hat{\mu}_{x(ST1)}, \hat{\mu}_{x(EH)}) = \frac{4\left[C_x^2 + C_\gamma^2(1+C_x^2)\right]}{[(C_x^2+C_{P1}^2)(1+C_x^2)]} \times 100\% \quad (2.8)$$

and

$$PRE(\hat{\mu}_{x(ST1)}, \hat{\mu}_{x(BBB)}) = \frac{4\left[C_x^2 + C_P^2(1+C_x^2)\right]}{[(C_x^2+C_{P1}^2)(1+C_x^2)]} \times 100\% \quad (2.9)$$

for different values of $C_\gamma, C_x, \theta$ and $P$. Findings are shown in Tables 1 and 2.

In the next section, we give a modified version of BBB randomized response model and discuss its properties.

## 3. The More General randomized response model

In the proposed randomized response model, the distribution of the responses is given by

$$Z_i = \begin{cases} \frac{(1-\alpha)X_i S}{\theta(1-P)} & \text{with probability } y\ (1-P) \\ \frac{\alpha X_i}{P} & \text{with probabiity } y\ P \end{cases} \tag{3.1}$$

where $0 \leqslant \alpha \leqslant 1$ is a known constant, and $P$ is neither equal to 'zero' nor equal to 'unity' i.e. $0 < P < 1$ selected in the sample.

In other words; each respondent selected in the sample is requested to rotate a spinner unobserved by the interviewer, and if the spinner stops in the shaded area, then the respondent is requested to report the response on the sensitive variable, say $\frac{\alpha X_i}{P}$; and if the spinner stops in the non – shaded area, then the respondent is requested to report the scrambled response, say $\frac{(1-\alpha)X_i S}{\theta(1-P)}$, where $S$ is any scrambling variable and its distribution is assumed to be known, and $\alpha$(i.e.$0 \leqslant \alpha \leqslant 1$ ) is assumed to be known constant. Assume that $E(S) = \theta$ and $V(S) = \sigma_s$ are known. Let $P$ be the proportion of the shaded area of the spinner and $(1-P)$ be the non shaded area of the spinner as shown in Fig. 2.

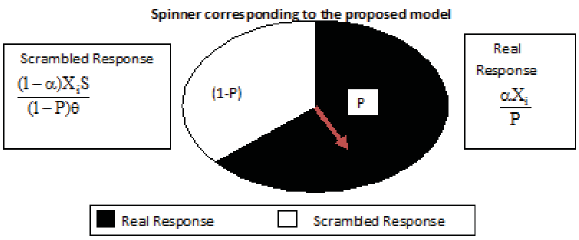


FIGURE 2.

From (2.1) we have

$$\begin{aligned} E(Z_i) &= \frac{(1-\alpha)E(X_i)E(S)(1-P)}{\theta(1-P)} + \frac{\alpha P E(X_i)}{P} \\ &= E(X_i)\left[\frac{(1-\alpha)\theta(1-P)}{\theta(1-P)} + \alpha\right] \\ &= E(X_i)(1-\alpha+\alpha) = E(X_i) = \mu_x \end{aligned}$$

Thus an unbiased estimator of $\mu_x$is given by

$$\hat{\mu}_{x(ST1)} = \bar{Z} = \frac{1}{n}\sum_{i=1}^{n} Z_i \tag{3.2}$$

The variance of $\hat{\mu}_{x(ST1)}$ is given by

$$V(\hat{\mu}_{x(ST1)}) = \frac{V(Z_i)}{n} \tag{3.3}$$

The variance of $Z_i$ is observed as follows:

$$\begin{aligned}
V(Z_i) &= \left[E(Z_i^2) - (E(Z_i))^2\right] \\
&= E(Z_i^2) - \mu_x^2 \\
&= \frac{(1-P)(1-\alpha^2)E(X_i^2)E(S^2)}{\theta^2(1-P)^2} + \frac{P\alpha^2 E(X_i^2)}{P^2} - \mu_x^2 \\
&= \mu_x^2\left[\frac{(1-\alpha)^2(1+C_x^2)(1+C_\gamma^2)}{(1-P)} + \frac{\alpha^2(1+C_x^2)}{P} - 1\right] \\
&= \mu_x^2\left[\alpha^2(1+C_x^2)\left\{\frac{1}{P} + \frac{(1+C_\gamma^2)}{(1-P)}\right\} - \frac{2a(1+C_x^2)(1+C_\gamma^2)}{(1-P)} + \frac{(1+C_x^2)(1+C_\gamma^2)}{(1-P)} - 1\right] \\
&= \mu_x^2\left[\frac{\alpha^2(1+C_x^2)(1+pP_\gamma^2)}{(1-P)P} - \frac{2a(1+C_x^2)(1+C_\gamma^2)}{(1-P)} + \frac{(1+C_x^2)(1+C_\gamma^2)}{(1-P)} - 1\right]
\end{aligned}$$

Thus the variance of $\hat{\mu}_{x(ST1)}$ is given by

$$\begin{aligned}
V(\hat{\mu}_{xo(ST1)}) &= \frac{\mu_x^2}{n}\left[\frac{(1+C_x^2)}{(1-P)}\left\{\frac{\alpha^2(1+PC_\gamma^2)}{P} - 2\alpha(1+C_\gamma^2) + (1+C_\gamma^2)\right\} - 1\right] \\
&= \frac{\mu_x^2}{n}\left[\frac{(1+C_x^2)}{(1-P)}\left\{\alpha^2(1-P) + P(1-\alpha)^2(1+C_\gamma^2)\right\} - 1\right]
\end{aligned} \tag{3.4}$$

The variance of $\hat{\mu}_{x(ST1)}$ is minimum when

$$\alpha = \frac{P(1+C_\gamma^2)}{(1+PC_\gamma^2)} = \alpha_0 \tag{3.5}$$

Substitution of (2.5) in (2.1) we get the optimum randomized response model as

$$Z_i = \begin{cases} \frac{X_i S}{\theta(1+PC_\gamma^2)} & \text{with probability } (1-P) \\ \frac{X_i(1+C_\gamma^2)}{(1+PC_\gamma^2)} & \text{with probability } P \end{cases} \tag{3.6}$$

Thus an unbiased estimator of $\mu_x$is given by

$$\hat{\mu}_{xo(ST1)} = \overline{Z}_0 = \frac{1}{n}\sum_{i=1}^{n} Z_{oi} \tag{3.7}$$

Putting (2.5) in (2.4) we get the minimum variance of $\hat{\mu}_{x(ST1)}$ (or the variance of the optimum estimator $\hat{\mu}_{xo(ST)}$ ) as "

$$\begin{aligned}
\min V(\hat{\mu}_{x(ST1)}) &= \frac{\mu_x^2}{n}\left[\frac{(1+C_x^2)(1+C_\gamma^2)}{(1+PC_\gamma^2)} - 1\right] \\
&= \frac{\mu_x^2}{n}\left[C_x^2 + (1+C_x^2)\left\{\frac{(1+C_\gamma^2)}{(1+PC_\gamma^2)}\right\} - 1\right] \\
&= \frac{\mu_x^2}{n}\left[C_x^2 + (1+C_x^2)C_P^{*2}\right] = V(\hat{\mu}_{xo(ST)})
\end{aligned} \tag{3.8}$$

where $C_P^{*2} = \left[\frac{(1+C_\gamma^2)}{(1+PC_\gamma^2)} - 1\right]$

### 3.1. Efficiency Comparison

From (1.3) ,(1.7) and (2.8) we have

$$V(\hat{\mu}_{x(EH)}) - V(\hat{\mu}_{xo(ST)}) = \frac{\mu_x^2(1+C_x^2)(1+C_\gamma^2)PC_\gamma^2}{n(1+PC_\gamma^2)} > 0 \tag{3.9}$$

$$V(\hat{\mu}_{x(BBB)}) - V(\hat{\mu}_{xo(ST)}) = \frac{\mu_x^2(1+C_x^2)}{n}\left(C_P^2 - C_P^{*2}\right) > 0$$

if

$$C_P^2 > C_P^{*2} \tag{3.10}$$

It follows from (3.9) that the proposed optimum estimator $\hat{\mu}_{xo(ST)}$ is always better than Eichhorn and Hayre (1983) estimator $\hat{\mu}_{x(EH)}$. It is also better than Bar – Lev et al. (2004) estimator $\hat{\mu}_{x(BBB)}$ as long as the condition is satisfied.

It is interesting to note that from (3.5)-(3.7) that the proposed optimum estimator is free from any kind of limitation as it depends only on known quantities such as $\theta, P$ and . Thus the proposed optimum estimator $\hat{\mu}_{xo(ST)}$ can be used in practice without any reservations.

**4. Numerical illustration** In order to judge the merits of the proposed estimator $\hat{\mu}_{xo(ST)}$ over $\hat{\mu}_{x(EH)}$ and $\hat{\mu}_{x(BBB)}$ we have computed the percent relative efficiency (PRE) of $\hat{\mu}_{xo(ST)}$ with respect to $\hat{\mu}_{x(EH)}$ and $\hat{\mu}_{x(BBB)}$ by using the formulae:

$$V(\hat{\mu}_{xo(ST)}, V(\hat{\mu}_{x(EH)}) = \frac{\left[C_x^2 + C_\gamma^2(1+C_x^2)\right]}{\left[C_x^2 + C_P^{*2}(1+C_x^2)\right]} \times 100\% \tag{4.1}$$

$$V(\hat{\mu}_{xo(ST)}, V(\hat{\mu}_{x(EH)}) = \frac{\left[C_x^2 + C_P^2(1+C_x^2)\right]}{\left[C_x^2 + C_P^{*2}(1+C_x^2)\right]} \times 100\% \tag{4.2}$$

for different values of $C_\gamma, C_x, \theta$ and $P$ . Findings are displayed in Table 3 and 4.

Tables 1 and 2 exhibit that the values of PRE $(\hat{\mu}_{xo(ST1)}, \hat{\mu}_{x(EH)})$and PRE $(\hat{\mu}_{xo(ST1)}, \hat{\mu}_{x(BBB)})$ are much larger than 100%. It follows that the proposed procedure $\hat{\mu}_{xo(ST1)}$ is more more efficient than Eichhorn and Hayre's (1983) estimator $\hat{\mu}_{x(EH)}$ and Bar Lev et al.'s (2004) estimator $\hat{\mu}_{x(BBB)}$. Thus our recommendation is to use the proposed estimator $\hat{\mu}_{xo(ST1)}$ in practice.

TABLE 1. The PRE($\hat{\mu}_{x(ST1)}$,$\hat{\mu}_{x(EH)}$)

| $\theta$ | $P$ | $C_\gamma$ | $C_x$ | PRE |
|---|---|---|---|---|
| 20.00 | 0.10 | 5.00 | 0.10 | 263.97 |
| 40.00 | 0.10 | 5.55 | 0.25 | 278.06 |
| 60.00 | 0.10 | 6.00 | 0.50 | 287.81 |
| 80.00 | 0.10 | 6.50 | 0.75 | 296.85 |
| 20.00 | 0.20 | 5.00 | 0.10 | 274.01 |
| 40.00 | 0.20 | 5.55 | 0.25 | 281.76 |
| 60.00 | 0.20 | 6.00 | 0.50 | 287.02 |
| 80.00 | 0.20 | 6.50 | 0.75 | 291.74 |
| 20.00 | 0.30 | 5.00 | 0.10 | 253.35 |
| 40.00 | 0.30 | 5.55 | 0.25 | 258.12 |
| 60.00 | 0.30 | 6.00 | 0.50 | 261.42 |
| 80.00 | 0.30 | 6.50 | 0.75 | 264.33 |
| 20.00 | 0.40 | 5.00 | 0.10 | 223.09 |
| 40.00 | 0.40 | 5.55 | 0.25 | 226.24 |
| 60.00 | 0.40 | 6.00 | 0.50 | 228.51 |
| 80.00 | 0.40 | 6.50 | 0.75 | 230.50 |
| 20.00 | 0.50 | 5.00 | 0.10 | 188.72 |
| 40.00 | 0.50 | 5.55 | 0.25 | 190.90 |
| 60.00 | 0.50 | 6.00 | 0.50 | 192.55 |
| 80.00 | 0.50 | 6.50 | 0.75 | 193.99 |
| 20.00 | 0.60 | 5.00 | 0.10 | 152.32 |
| 40.00 | 0.60 | 5.55 | 0.25 | 153.86 |
| 60.00 | 0.60 | 6.00 | 0.50 | 155.09 |
| 80.00 | 0.60 | 6.50 | 0.75 | 156.15 |
| 20.00 | 0.70 | 5.00 | 0.10 | 114.85 |
| 40.00 | 0.70 | 5.55 | 0.25 | 115.92 |
| 60.00 | 0.70 | 6.00 | 0.50 | 116.81 |
| 80.00 | 0.70 | 6.50 | 0.75 | 117.58 |

TABLE 2. The PRE($\hat{\mu}_{x(ST1)}$,$\hat{\mu}_{x(BBB)}$)

| $\theta$ | $P$ | $C_\gamma$ | $C_x$ | PRE |
|---|---|---|---|---|
| 20.00 | 0.10 | 5.00 | 0.10 | 291.10 |
| 40.00 | 0.10 | 5.55 | 0.25 | 308.14 |
| 60.00 | 0.10 | 6.00 | 0.50 | 319.29 |
| 80.00 | 0.10 | 6.50 | 0.75 | 329.40 |
| 20.00 | 0.20 | 5.00 | 0.10 | 336.49 |
| 40.00 | 0.20 | 5.55 | 0.25 | 349.86 |
| 60.00 | 0.20 | 6.00 | 0.50 | 357.32 |
| 80.00 | 0.20 | 6.50 | 0.75 | 363.46 |
| 20.00 | 0.30 | 5.00 | 0.10 | 350.63 |
| 40.00 | 0.30 | 5.55 | 0.25 | 364.11 |
| 60.00 | 0.30 | 6.00 | 0.50 | 370.53 |
| 80.00 | 0.30 | 6.50 | 0.75 | 375.25 |
| 20.00 | 0.40 | 5.00 | 0.10 | 353.19 |
| 40.00 | 0.40 | 5.55 | 0.25 | 369.04 |
| 60.00 | 0.40 | 6.00 | 0.50 | 375.71 |
| 80.00 | 0.40 | 6.50 | 0.75 | 380.06 |
| 20.00 | 0.50 | 5.00 | 0.10 | 348.46 |
| 40.00 | 0.50 | 5.55 | 0.25 | 368.68 |
| 60.00 | 0.50 | 6.00 | 0.50 | 376.56 |
| 80.00 | 0.50 | 6.50 | 0.75 | 381.23 |
| 20.00 | 0.60 | 5.00 | 0.10 | 336.59 |
| 40.00 | 0.60 | 5.55 | 0.25 | 363.57 |
| 60.00 | 0.60 | 6.00 | 0.50 | 373.77 |
| 80.00 | 0.60 | 6.50 | 0.75 | 379.44 |
| 20.00 | 0.70 | 5.00 | 0.10 | 314.69 |
| 40.00 | 0.70 | 5.55 | 0.25 | 351.97 |
| 60.00 | 0.70 | 6.00 | 0.50 | 366.16 |
| 80.00 | 0.70 | 6.50 | 0.75 | 373.83 |

TABLE 3. The PRE($\hat{\mu}_{xo(ST)}$,$\hat{\mu}_{x(EH)}$)

| $\theta$ | $P$ | $C_\gamma$ | $C_x$ | PRE |
|---|---|---|---|---|
| 20.00 | 0.10 | 1.00 | 0.15 | 121.64 |
| 40.00 | 0.10 | 1.30 | 0.20 | 171.82 |
| 60.00 | 0.10 | 1.50 | 0.25 | 209.96 |
| 80.00 | 0.10 | 1.70 | 3.00 | 190.01 |
| 20.00 | 0.20 | 1.00 | 0.15 | 148.40 |
| 40.00 | 0.20 | 1.30 | 0.25 | 224.84 |
| 60.00 | 0.20 | 1.50 | 0.35 | 283.12 |
| 80.00 | 0.20 | 1.70 | 0.40 | 356.66 |
| 20.00 | 0.30 | 1.00 | 0.15 | 182.35 |
| 40.00 | 0.30 | 1.30 | 0.25 | 298.93 |
| 60.00 | 0.30 | 1.50 | 0.35 | 392.10 |
| 80.00 | 0.30 | 1.70 | 0.40 | 518.39 |
| 20.00 | 0.40 | 1.00 | 0.15 | 226.82 |
| 40.00 | 0.40 | 1.30 | 0.25 | 405.63 |
| 60.00 | 0.40 | 1.50 | 0.35 | 555.19 |
| 80.00 | 0.40 | 1.70 | 0.40 | 775.89 |
| 20.00 | 0.50 | 1.00 | 0.15 | 287.61 |
| 40.00 | 0.50 | 1.30 | 0.25 | 572.57 |
| 60.00 | 0.50 | 1.50 | 0.35 | 826.02 |
| 80.00 | 0.50 | 1.70 | 0.40 | 1250.05 |
| 20.00 | 0.60 | 1.00 | 0.15 | 375.73 |
| 40.00 | 0.60 | 1.30 | 0.25 | 870.80 |
| 60.00 | 0.60 | 1.50 | 0.35 | 1363.97 |
| 80.00 | 0.60 | 1.70 | 0.40 | 2412.79 |
| 20.00 | 0.70 | 1.00 | 0.15 | 514.93 |
| 40.00 | 0.70 | 1.30 | 0.25 | 1555.62 |
| 60.00 | 0.70 | 1.50 | 0.35 | 2948.72 |
| 80.00 | 0.70 | 1.70 | 0.40 | 9741.27 |

TABLE 4. The PRE($\hat{\mu}_{xo(ST)}$,$\hat{\mu}_{x(BBB)}$)

| $\theta$ | $P$ | $C_\gamma$ | $C_x$ | PRE |
|---|---|---|---|---|
| 20.00 | 0.10 | 1.00 | 0.15 | 145.21 |
| 40.00 | 0.10 | 1.30 | 0.20 | 199.90 |
| 60.00 | 0.10 | 1.50 | 0.25 | 241.59 |
| 80.00 | 0.10 | 1.70 | 3.00 | 211.08 |
| 20.00 | 0.20 | 1.00 | 0.15 | 212.21 |
| 40.00 | 0.20 | 1.30 | 0.25 | 305.97 |
| 60.00 | 0.20 | 1.50 | 0.35 | 376.60 |
| 80.00 | 0.20 | 1.70 | 0.40 | 467.66 |
| 20.00 | 0.30 | 1.00 | 0.15 | 314.38 |
| 40.00 | 0.30 | 1.30 | 0.25 | 482.20 |
| 60.00 | 0.30 | 1.50 | 0.35 | 612.72 |
| 80.00 | 0.30 | 1.70 | 0.40 | 793.72 |
| 20.00 | 0.40 | 1.00 | 0.15 | 476.36 |
| 40.00 | 0.40 | 1.30 | 0.25 | 787.92 |
| 60.00 | 0.40 | 1.50 | 0.35 | 1037.30 |
| 80.00 | 0.40 | 1.70 | 0.40 | 1413.12 |
| 20.00 | 0.50 | 1.00 | 0.15 | 747.08 |
| 40.00 | 0.50 | 1.30 | 0.25 | 1368.79 |
| 60.00 | 0.50 | 1.50 | 0.35 | 1890.15 |
| 80.00 | 0.50 | 1.70 | 0.40 | 2777.30 |
| 20.00 | 0.60 | 1.00 | 0.15 | 1234.09 |
| 40.00 | 0.60 | 1.30 | 0.25 | 2643.36 |
| 60.00 | 0.60 | 1.50 | 0.35 | 3956.73 |
| 80.00 | 0.60 | 1.70 | 0.40 | 6780.20 |
| 20.00 | 0.70 | 1.00 | 0.15 | 2208.84 |
| 40.00 | 0.70 | 1.30 | 0.25 | 6287.72 |
| 60.00 | 0.70 | 1.50 | 0.35 | 11435.00 |
| 80.00 | 0.70 | 1.70 | 0.40 | 36614.91 |

Tables 3 and 4 show that the values of PRE ($\hat{\mu}_{xo(ST)}, \hat{\mu}_{x(EH)}$)and PRE ($\hat{\mu}_{xo(ST)}, \hat{\mu}_{x(BBB)}$) are much greater than 100%. So, we state that the proposed optimum estimator $\hat{\mu}_{xo(ST)}$ is more efficient than $\hat{\mu}_{x(EH)}$ and $\hat{\mu}_{x(BBB)}$ with considerable gain in efficiency. Thus, based on our simulation results, the use of the proposed estimator $\hat{\mu}_{xo(ST)}$ is recommended for its use in practice.

**5. Conclusion** Utilizing the idea of obtaining a response from each respondent, a new class of estimators $\hat{\mu}_{x(ST1)}$ and $\hat{\mu}_{xo(ST)}$ has been proposed. We have obtained the variance of the proposed class of estimators $\hat{\mu}_{x(ST1)}$ and $\hat{\mu}_{xo(ST)}$ and compared with Eichhorn and Hayre's (1983) estimator $\hat{\mu}_{x(EH)}$ and Bar – Lev et al.'s (2004) estimator $\hat{\mu}_{x(BBB)}$ . It has been found that the proposed class of estimators $\hat{\mu}_{x(ST1)}$ and $\hat{\mu}_{xo(ST)}$ are more efficient than $\hat{\mu}_{x(EH)}$ and$\hat{\mu}_{x(BBB)}$ under very realistic conditions. We have also shown numerically that the proposed estimators $\hat{\mu}_{x(ST1)}$ and $\hat{\mu}_{xo(ST)}$ are better than $\hat{\mu}_{x(EH)}$ and$\hat{\mu}_{x(BBB)}$ respectively. Thus our recommendation is to use the proposed estimators $\hat{\mu}_{x(ST1)}$ and $\hat{\mu}_{xo(ST)}$ instead of $\hat{\mu}_{x(EH)}$ and$\hat{\mu}_{x(BBB)}$ in practice.

## 6. Acknowledgements

The authors are thankful to the Editor - in - Chief, and to the anonymous learned referee for his valuable suggestions regarding improvement of the paper.

### References

[1] Barabesi, L., Diana, G. and Perri, P.F. (2014). Horvitz-Thompson estimation with randomized response and non-response. *Model Assist. Statist. Appl.*,9(1), 3-10.

[2] Bar –lev, S.K., Bobovitch, E. and Boukai, B.(2004). A note on Randomized response models for quantitative data. *Metrika*, 60, 225-250.

[3] Eichhorn, B.H. and Hayre, L.S. (1983). Scrambled randomized response methods for obtaining sensitive quantitative data. *Jour. Statist. Plann. Infer.*, 7,307-316.

[4] Fox, J.A. and Tracy, P.E. (1986). Randomized Response: A method of Sensitive Surveys. Newbury Park, CA: SEGE Publications.

[5] Grewal, I.S., Bansal, M.L., and Sidhu, S.S. (2005–2006). Population mean corresponding to Horvitz–Thompson's estimator for multi-characteristics using randomized response technique. *Model Assist. Statist. Appl.* 1, 215-220.

[6] Hong, Z. (2005–2006). Estimation of mean in randomized response surveys when answers are incompletely truthful. *Model Assist. Statist. Appl.*, 1,221-230.

[7] Mahajan, P.K., Sharma, P. and Gupta, R.K. (2007). Optimum stratification for allocation proportional to strata totals for scrambled response. *Model Assist. Statist. Appl.*, 2(2), 81-88.

[8] Odumade, O. and Singh, S. (2009). Improved Bar-Lev, Bobovitch, and Boukai Randomized Response Models. *Commun. Statist. Simul. Comput.*, 38, 473-502.

[9] Odumade, O. and Singh, S. (2010). An alternative to the Bar-Lev, Bobovitch, and Boukai Randomized Response *Model. Sociol. Meth. Res.*, 39(2), 206-221.

[10] Perri, P.F. (2008). Modified randomized devices for Simmons' model. *Model Assist. Statist. Appl.*, 3(3), 233-239.

[11] Ryu, J.B., Kim, J.M., Heo, T.Y. and Park, C.G. (2005–2006). On stratified randomized response sampling. *Model Assist. Statist. Appl.*, 1, 31-36.

[12] Singh, S. and Cheng, S.C. (2009). Utilization of higher order moments of scrambling variables in randomized response sampling. *Jour. Statist. Plann. Infer.*, 139, 3377-3380.

[13] Singh, H. P. and Tarray, T. A. (2012). A Stratified Unknown repeated trials in randomized response sampling. *Commun. Korean Statist. Soc.*, 19, (6), 751-759.

[14] Singh, H.P. and Tarray, T.A.(2013). A modified survey technique for estimating the proportion and sensitivity in a dichotomous finite population. *Int. Jour. Adv. Sci. and Tech. Res.*, 3(6), 459 - 472.

[15] Singh, H.P. and Tarray, T.A. (2014). A dexterous randomized response model for estimating a rare sensitive attribute using Poisson distribution. *Statist. Prob. Lett.*, 90,42-45.

[16] Warner, S.L.(1965). Randomized response: A survey technique for eliminating evasive answer bias. *Jour. Amer. Statist. Assoc.*,60,63-69.