İSTATİSTİK: JOURNAL OF THE TURKISH STATISTICAL ASSOCIATION

İSTATİSTİK

Vol. 7, No. 2, July 2014, pp. 55–62

ISSN 1300-4077 | 14 | 2 | 55 | 62

ESTIMATIONS FOR PROPORTIONAL REVERSED HAZARD RATE MODEL DISTRIBUTIONS BASED ON UPPER RECORD VALUES AND INTER-RECORD TIMES

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Abstract: The distributions from proportional reversed hazard rate models have been extensively studied in literature because of their wide applicability in the modelling and analysis of lifetime data. Moreover, record values and their properties have also been examined by many authors. In this study, we consider the distributions from the two-parameter proportional reversed hazard rate models. The maximum likelihood and Bayes estimates are obtained for the unknown parameters based on upper record values with the number of trials following the record values (inter-record times). These estimates are compared in terms of the estimated risk by the Monte Carlo simulations for the generalized Rayleigh (Burr Type X) distribution.

Key words: Bayes estimation; Inter-record times; PRHR model; Record values.

History: Submitted: 4 September 2014; Revised: 25 October 2014; Accepted: 28 October 2014

1. Introduction

Let $X_1, X_2, ...$ be a sequence of independent and identically distributed (iid) random variables with cumulative distribution function (cdf) $F(x;\theta)$ and probability density function (pdf) $f(x;\theta)$. The observation X_j is an upper record value of $\{X_m,\ m \geq 1\}$ if it is greater than all the preceding observations in other words, if we define $Y_m = \max(X_1, ..., X_m)$, $m \geq 1$ then, X_j is an upper record value if $Y_j > Y_{j-1}$, $j > 1$. The record times are the indices at which upper record values occur. The record time sequence for upper record values $\{U(m),\ m \geq 1\}$ is defined in the following manner: $U(1) = 1$ with probability 1 and mth record time, for $m > 1$

$$U(m) = \min\left\{ j : j > U(m-1),\ X_j > X_{U(m-1)} \right\}.$$

Then, the sequence $\left\{X_{U(m)},\ m \geq 1\right\}$ and $\{U(m),\ m \geq 1\}$ represent an upper record values and corresponding record times, respectively. By definition, X_1 is an upper record value. An analogous definition can be provided for the lower record values. Inter-record times K_i is defined by $K_i = U(i+1) - U(i)$, $i = 1, 2, ...$ In another words, K_i is the number of trials following the observation of $X_{U(i)}$ that are needed to obtain a new record value, say $X_{U(i+1)}$. It corresponds roughly to the number of non-record observations between record values.

Record values and the associated statistics are of interest in many real life problems involving weather, sports, economics, life-tests and so on. In recent years there has been a growing interest in the study of inference problems associated with record values. For example, the Bayesian estimation

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for the two parameters of some life distributions, including exponential, Weibull, Pareto and Burr Type XII, based on upper record values were considered by Ahmadi and Doostparast [1]. The different point estimates and prediction of future record values for the unknown parameters of exponentiated family of distributions based on lower record values were derived by Asgharzadeh and Fallah [2]. The Bayesian and non-Bayesian estimations of the parameters as well as survival and hazard functions for a class of an exponential family based on upper record values presented by Wang and Shi [3]. When the underlying distribution is generalized exponential distribution, non-Bayesian and Bayesian point estimates as well as confidence intervals for the unknown parameters based on upper record values with their inter-record times were constructed by Kızılaslan and Nadar [4]. For more detailed references about the record values see Arnold et al. [5].

Let the continuous random variable X have the cdf in the form of $F(x;\theta) = [F_0(x)]^{\theta}$ where $F_0(x)$ is the baseline cdf which is independent of the shape parameter θ and $\theta > 0$. This family of distributions is well known in the literature as proportional reversed hazard rate (PRHR) model and is equivalent to say that $r_F(x;\theta) = \theta r_{F_0}(x;\theta)$ where r_F is the corresponding reversed hazard rate function of the cdf F with the pdf f which is defined as

$$r_F(x;\theta) = \frac{d}{dx}\ln F(x;\theta) = \frac{f(x;\theta)}{F(x;\theta)}.$$

In the literature, this family of distributions is also defined as F^{θ} distribution. When θ is a positive integer, F^{θ} is also defined as Lehmann alternatives, see Lehmann [6]. Many researchers and authors have developed various classes of F^{θ} distributions. Some distributional properties of order statistics and record values from F^{θ} distributions were reviewed by Shakil and Ahsanullah [7]. The PRHR models consist of several well known distributions such as Burr Type III, generalized Rayleigh (Burr Type X), Type I generalized logistic distribution, Fréchet, exponentiated Weibull, generalized exponential, power function and so on.

The cdf and pdf of the two-parameter PRHR model is

$$F_X(x;\theta,\sigma) = [F_0(x/\sigma)]^{\theta},\ x \in B,\ \theta > 0,\ \sigma > 0, \tag{1.1}$$

$$f_X(x;\theta,\sigma) = \frac{\theta}{\sigma} f_0(x/\sigma)\,[F_0(x/\sigma)]^{\theta-1},\ x \in B,\ \theta > 0,\ \sigma > 0, \tag{1.2}$$

where $F_0(.)$ and $f_0(.)$ are a baseline cdf and pdf, B is the support of the continuous random variable X, which is independent of the parameter θ, θ and σ are the shape and scale parameters, respectively (see Asgharzadeh and Fallah [2]). The point estimates of θ and σ for the two-parameter PRHR model based on lower records under the classical and Bayesian frameworks were obtained by Asgharzadeh and Fallah [2], but the inter-record times was not taken into consideration. They also derived the prediction of the future record values from Bayesian view point and a Monte Carlo simulation is performed for the generalized exponential distribution. In this study, the point estimates of θ and σ for the two-parameter PRHR model are obtained based on the upper record values and their corresponding inter-record times under the classical and Bayesian frameworks. A Monte Carlo simulation is performed for the generalized Rayleigh distribution.

The paper is organized as follows. In Section 2, we derive maximum likelihood estimation (MLE) of the parameters under the inverse sampling scheme. In Section 3, we obtain the Bayes estimates for the parameters under the symmetric and asymmetric loss functions. In Section 4, a computer simulation study is performed to compare the different estimators for the generalized Rayleigh distribution and the results are reported. Finally, we conclude the paper in Section 5.

2. Non-Bayesian analysis

Under the inverse sampling scheme units are taken sequentially and the sampling is terminated when the mth maximum observation is obtained. In this case, the total number of units sampled is a random number, and K_m is defined to be one for convenience.

In this section, we consider the parameter estimation for the two-parameter PRHR model distributions based on upper record values with their corresponding inter-record times under the inverse sampling scheme.

Let $X_1, X_2, ...$ be iid random variables each drawn from a population with cdf $F(.)$ and pdf $f(.)$. Then the likelihood function associated with the sequence $(R_1, K_1, R_2, K_2, ..., R_m, K_m)$ is given in Hofmann and Nagaraja [8] as

$$L(\mathbf{r}, \mathbf{k}) = \prod_{i=1}^{m} f(r_i) \{F(r_i)\}^{k_i - 1} I_{(r_{i-1}, \infty)}(r_i), \tag{2.1}$$

where $r_0 \equiv -\infty$, $k_m \equiv 1$ and $I_A(x)$ is the indicator function of the set A.

We assume that the sequence $(R_1, K_1, R_2, K_2, ..., R_m, K_m)$ is arising from the two-parameter PRHR model with parameters θ and σ. Then, from the Equations (1.1)-(2.1), we have

$$L(\theta, \sigma; \mathbf{r}, \mathbf{k}) = \frac{\theta^m}{\sigma^m} \exp\left\{ \sum_{i=1}^{m} \ln f_0(r_i/\sigma) + \sum_{i=1}^{m} (\theta k_i - 1) \ln F_0(r_i/\sigma) \right\}, \ r_1 < ... < r_m. \tag{2.2}$$

Then, the MLEs of θ and σ are given by

$$\widehat{\theta} = \frac{m}{\sum_{i=1}^{m} k_i \ln F_0(r_i/\widehat{\sigma})}, \tag{2.3}$$

and $\widehat{\sigma}$ is the solution of the following nonlinear equation

$$-\frac{m}{\widehat{\sigma}} - \frac{1}{\widehat{\sigma}^2} \sum_{i=1}^{m} \frac{r_i f_0'(r_i/\widehat{\sigma})}{f_0(r_i/\widehat{\sigma})} + \frac{m}{\widehat{\sigma}^2 \sum_{i=1}^{m} k_i \ln F_0(r_i/\widehat{\sigma})} \sum_{i=1}^{m} \frac{k_i r_i f_0(r_i/\widehat{\sigma})}{F_0(r_i/\widehat{\sigma})} + \frac{1}{\widehat{\sigma}^2} \sum_{i=1}^{m} \frac{r_i f_0(r_i/\widehat{\sigma})}{F_0(r_i/\widehat{\sigma})} = 0. \tag{2.4}$$

For a special case, when the $F_0(x) = 1 - e^{-x^2}$, $x > 0$, X has the generalized Rayleigh (Burr Type X) distribution. Therefore, the MLE of θ is

$$\widehat{\theta} = \frac{m}{\sum_{i=1}^{m} k_i \ln\left(1 - e^{-(r_i/\widehat{\sigma})^2}\right)}, \tag{2.5}$$

and the MLE of σ, $\widehat{\sigma}$ is the solution of

$$-\frac{m}{\widehat{\sigma}} + \frac{1}{\widehat{\sigma}^3} \sum_{i=1}^{m} r_i^2 + \frac{m}{\widehat{\sigma}^3 \sum_{i=1}^{m} k_i \ln\left(1 - e^{-(r_i/\widehat{\sigma})^2}\right)} \sum_{i=1}^{m} \frac{k_i r_i^2 e^{-(r_i/\widehat{\sigma})^2}}{1 - e^{-(r_i/\widehat{\sigma})^2}} + \frac{1}{\widehat{\sigma}^3} \sum_{i=1}^{m} \frac{r_i^2 e^{-(r_i/\widehat{\sigma})^2}}{1 - e^{-(r_i/\widehat{\sigma})^2}} = 0. \tag{2.6}$$

It is clear that the nonlinear equation (2.6) can be solved by using numerical methods such as fixed point iteration, Newton-Raphson.

3. Bayesian analysis

Bayesian approach has a number of advantages over the conventional frequentist approach. Bayes theorem is the only consistent way to modify our beliefs about the parameters given the data that actually occurred. In this section, we consider the Bayes estimates of the parameters for two-parameter PRHR model distributions based on upper record values with their corresponding inter-record times under different loss functions.

In the Bayesian inference, the most commonly used loss function is the squared error loss (SEL), $L(\theta^*,\theta)=(\theta^*-\theta)^2$, where θ^* is an estimate of θ. This loss function is symmetrical and gives equal weight to overestimation as well as underestimation. It is well known that the use of symmetric loss functions may be inappropriate in many circumstances, particularly when positive and negative errors have different consequences. One of the most popular asymmetric loss function is the linear-exponential (LINEX) loss, $L(\theta^*,\theta)=e^{v(\theta^*-\theta)}-v(\theta^*-\theta)-1$, $v\neq 0$, which was introduced by Varian [9]. The sign and magnitude of v represents the direction and degree of asymmetry, respectively. For v close to zero, the LINEX loss is approximately the SEL and therefore almost symmetric.

To use a general joint prior for θ and σ causes some computational complexities for the Bayes estimates of the parameters based on the records with their corresponding inter-record times from the PRHR models, when the parameters θ and σ are assumed to be unknown. To overcome the computational obstacle, we use Soland's method (see Soland [10]) in which he considered a family of joint prior distributions that places continuous distributions on the scale parameter and discrete distributions on the shape parameter.

In our case, for the scale parameter σ it is assumed that, as in Asgharzadeh and Fallah [2], the parameter σ is restricted to a finite number of values $\sigma_1,...,\sigma_k$ with the probabilities $\eta_1,...,\eta_k$, respectively, where $0\leq\eta_j\leq 1$ and $\sum_{j=1}^k\eta_j=1$, i.e. $\pi(\sigma_j)=P(\sigma=\sigma_j)=\eta_j,\ j=1,...,k$. Further, suppose that conditional upon $\sigma=\sigma_j$, θ has a natural conjugate prior with distribution having gamma (a_j,b_j) with density

$$\pi(\theta|\,\sigma_j)=\frac{b_j^{a_j}\theta^{a_j-1}e^{-\theta b_j}}{\Gamma(a_j)},\ a_j,b_j>0. \tag{3.1}$$

Combining the likelihood function in Equation (2.2) and the prior density function Equation (3.1), we obtain the conditional posterior distribution of θ is given by

$$\pi^*(\theta|\,\sigma_j;\mathbf{r},\mathbf{k})=\frac{L(\theta,\sigma_j;\mathbf{r},\mathbf{k})\pi(\theta|\,\sigma_j)}{\int_0^\infty L(\theta,\sigma_j;\mathbf{r},\mathbf{k})\pi(\theta|\,\sigma_j)d\theta}=\frac{B_j^{A_j}\theta^{A_j-1}e^{-\theta B_j}}{\Gamma(A_j)}, \tag{3.2}$$

where $A_j=m+a_j$ and $B_j=b_j-\sum_{i=1}^m k_i\ln F_0(r_i/\sigma_j),\ j=1,...,k$. The joint posterior density function of θ and σ_j is given by

$$\pi^*(\theta,\sigma_j|\,\mathbf{r},\mathbf{k})=\frac{L(\theta,\sigma_j;\mathbf{r},\mathbf{k})\pi(\theta|\,\sigma_j)\pi(\sigma_j)}{\sum_{j=1}^k\int_0^\infty L(\theta,\sigma_j;\mathbf{r},\mathbf{k})\pi(\theta|\,\sigma_j)\pi(\sigma_j)d\theta}=\frac{b_j^{a_j}u_j\eta_j\theta^{A_j-1}e^{-\theta B_j}}{A\Gamma(a_j)\sigma_j^m} \tag{3.3}$$

where $u_j=\prod_{i=1}^m(f_0(r_i/\sigma_j)/F_0(r_i/\sigma_j))$ and A is a normalized constant,

$$A=\sum_{j=1}^k\left(b_j^{a_j}u_j\eta_j\Gamma(A_j)/\sigma_j^m B_j^{A_j}\Gamma(a_j)\right).$$

Furthermore, the marginal posterior density function of σ_j is

$$P_j=P(\sigma=\sigma_j|\,\mathbf{r},\mathbf{k})=\int_0^\infty\pi^*(\theta,\sigma_j|\,\mathbf{r},\mathbf{k})d\theta=\frac{b_j^{a_j}u_j\eta_j\Gamma(A_j)}{\sigma_j^m B_j^{A_j}\Gamma(a_j)A},\ j=1,...,k. \tag{3.4}$$

Then, the Bayes estimates of the parameters θ and σ under the SE loss function, say $\widehat{\theta}_{BS}$ and $\widehat{\sigma}_{BS}$, are given by

$$\widehat{\theta}_{BS}=\int_0^\infty\sum_{j=1}^k\theta P_j\pi^*(\theta|\,\sigma_j;\mathbf{r},\mathbf{k})d\theta=\sum_{j=1}^k\frac{P_jA_j}{B_j}, \tag{3.5}$$

and

$$\widehat{\sigma}_{BS}=\int_0^{\infty}\sum_{j=1}^{k}\sigma_jP_j\pi^*(\theta|\,\sigma_j;\mathbf{r},\mathbf{k})d\theta=\sum_{j=1}^{k}\sigma_jP_j. \tag{3.6}$$

The Bayes estimates of the parameters θ and σ under the LINEX loss function, say $\widehat{\theta}_{BL}$ and $\widehat{\sigma}_{BL}$, are given by

$$\widehat{\theta}_{BL}=-\frac{1}{v}\ln\left[\int_0^{\infty}\sum_{j=1}^{k}e^{-v\theta}P_j\pi^*(\theta|\,\sigma_j;\mathbf{r},\mathbf{k})d\theta\right]=-\frac{1}{v}\ln\left[\sum_{j=1}^{k}P_j\left(1+\frac{v}{B_j}\right)^{-A_j}\right], \tag{3.7}$$

and

$$\widehat{\sigma}_{BL}=-\frac{1}{v}\ln\left[\int_0^{\infty}\sum_{j=1}^{k}e^{-v\sigma_j}P_j\pi^*(\theta|\,\sigma_j;\mathbf{r},\mathbf{k})d\theta\right]=-\frac{1}{v}\ln\left[\sum_{j=1}^{k}P_je^{-v\sigma_j}\right]. \tag{3.8}$$

To implement the calculations, it is first necessary to draw the values of (σ_j,η_j) and the hyperparameters (a_j,b_j) in the conjugate prior for $j=1,...,k$. The former pairs of values are fairly straightforward to specify, but for (a_j,b_j) it is necessary to condition prior beliefs about θ on each σ_j in turn, and this can be difficult in practice. One of the useful methods to estimate the hyperparameters (a_j,b_j), $j=1,...,k$ is the moment approach.

Let $T_i=-\ln F_0(r_i/\sigma)$ are the record values from exponential distribution with conditional density $f_T(t;\theta)=\theta e^{-\theta t}$, $t>0$. Furthermore, for a given σ_j the marginal density function and cdf are given by

$$f_T(t)=\int_0^{\infty}\pi^*(\theta|\,\sigma_j;\mathbf{r},\mathbf{k})f_T(t;\theta)d\theta=\frac{b_j^{a_j}a_j}{(b_j+t)^{a_j+1}},\ t>0,$$

and

$$F_T(t)=1-\frac{b_j^{a_j}}{(b_j+t)^{a_j}},\ t>0.$$

It can be easily obtained that $E(T)=b_j/(a_j-1)$ and $E(T^2)=2b_j^2/(a_j-1)(a_j-2)$. Therefore, the moment estimates of a_j and b_j, $j=1,...,k$ are given by

$$a_j=\left(\frac{T_1}{2\overline{T}^2}-1\right)^{-1}+2,\ b_j=(a_j-1)\overline{T}, \tag{3.9}$$

where $\overline{T}=-\sum_{i=1}^{m}\left(\ln F_0(r_i/\sigma_j)\right)/m$ and $T_1=\sum_{i=1}^{m}\left(-\ln F_0(r_i/\sigma_j)\right)^2/m$.

For the generalized Rayleigh distribution, the Bayes estimates given in this section are obtained by using $F_0(x)=1-e^{-x^2}$, $x>0$.

4. Simulation study

In this section, we present some numerical results to compare the performance of the different methods for different sample sizes and different priors. The performance of the point estimators are compared by using the estimated risks (ERs). The estimated risk (ER) of θ, when θ is estimated by $\widehat{\theta}$, under the SEL function is given by $ER(\theta)=\sum_{i=1}^{N}(\widehat{\theta}_i-\theta_i)/N$ and under the LINEX loss function is given by $ER(\theta)=\sum_{i=1}^{N}\left(e^{v(\widehat{\theta}_i-\theta_i)}-v(\widehat{\theta}_i-\theta_i)-1\right)/N$, where N is the number of replication. All of the computations are performed by using Matlab R2010a. All the results are based on 1000 replications.

In Table 1, the ML and Bayes estimates under the SE and the LINEX loss functions with their corresponding ERs for the generalized Rayleigh distribution are listed when $\theta=2$ and $\sigma=3.5$. We assume that σ_j, $j=1,...,10$ takes the values: 3(0.1)3.9 with equal probability for $\sigma=3.5$. It is observed that as the sample size increases the estimated risk of the estimators generally decrease.

The performance of the Bayes estimates under the SEL function is better than MLEs. Since the performance of the Bayes estimates under the LINEX loss function depend on the asymmetry parameter v, some of these estimates is better than that of other estimates.

In Table 2, to see the effect of the inter-record times in ML estimates, the ML estimates of θ and σ with their corresponding ERs for the generalized Rayleigh distribution are listed when $(\theta, \sigma) = (2, 5)$ and $(6, 3)$. For the comparison purpose, we will generate the lower and the upper record values by using the following procedure.

Step 1. Firstly, we generate a random sample from the generalized Rayleigh distribution with sample size n.

Step 2. The lower record values and the upper record values with their corresponding inter-record times are saved. Notice that the sample sizes of the lower and the upper record values can be different.

Step 3. The ML estimates of θ and σ are computed based on lower record values.

Step 4. The ML estimates of θ and σ are computed based on upper record values and their corresponding inter-record times.

Step 5. Repeat Steps 1-4, 2000 times and obtain the samples (θ_i, σ_i), $i = 1, ..., N$.

Since the data is generated from the generalized Rayleigh distribution with fixed parameters, estimations should be obtained by using either lower or upper record values and these estimates should be close to each other.

Then, the average estimates with their corresponding ERs are computed and listed in Table 2. It is observed that as the sample size increases the estimated risk of the estimators decrease. The performance of the ML estimates based on the upper record values with their corresponding inter-record times is better than the one based on only lower record values.

5. Conclusion

It was observed that using the record values with the corresponding inter-record times reduces the estimated risk of the point estimates and the point predictors for the distributions considered in Kızılaslan and Nadar [4] and Nadar and Kızılaslan [11].

Asgharzadeh and Fallah [2] considered estimations based on only lower record values for the two-parameter proportional reversed hazard rate model distributions. However, if we want to use both lower record value and their corresponding inter-record times, the structure of the likelihood function yields a non-linear system of equations for the MLE of the parameters and the Bayesian estimates have difficulties. On the other hand, if we use the upper record values and their corresponding inter-record times, which is considered in this paper, then one of the MLE have a closed form and other is a solution of a non-linear equation. It should be underlined that the structure of the ML estimates for our case has much simpler than those obtained by using lower record values and their corresponding inter record times. Moreover, the Bayesian estimates of the parameters can be obtained easily than that of other case. Therefore, we suggest either to use the upper record values with their inter-record times or lower record values without considering their inter-record times for the proportional reversed hazard rate family distributions.

In the simulation part of this study, the different estimations for the shape and scale parameters for the two-parameter proportional reversed hazard rate model distributions based on upper record values with their corresponding inter-record times are obtained. Moreover, to see the effect of the inter-record times, the ML estimates are also obtained based on only lower record values. These estimates are compared by using estimated risks for the generalized Rayleigh distribution.

The simulation illustrates that the Bayesian estimates under the symmetric and asymmetric loss (for $v = -1, 1$) functions are preferable than others. Moreover, it is observed that using the inter-record times in the MLE case decrease the estimated risk.

TABLE 1. Estimates for θ and σ by using conjugate and discrete priors when the true values of $\theta = 2$ and $\sigma = 3.5$. The first and second rows represent the average estimates and the estimated risks, respectively.

		Bayes Estimates				
			LINEX			
	MLE	SEL	$v=-2$	$v=-1$	$v=1$	$v=2$
			$m=6$			
θ	2.8314	2.5768	6.7432	3.3904	2.1693	1.9036
	69.1065	1.8188	8.7686	0.9493	1.2760	5.0158
σ	3.4859	3.3904	3.4408	3.4156	3.3656	3.3417
	0.3118	0.0452	0.0952	0.0230	0.0224	0.0894
			$m=8$			
θ	2.2861	2.6611	7.8222	3.6296	2.2037	1.9268
	52.8758	1.7564	10.8369	1.0933	1.1237	5.0051
σ	3.5376	3.4071	3.4489	3.4280	3.3868	3.3670
	0.1485	0.0409	0.0832	0.0206	0.0203	0.0810
			$m=10$			
θ	1.9798	2.6898	8.9651	3.8111	2.2005	1.9155
	42.5549	1.6600	13.0888	1.2347	1.7819	5.8867
σ	3.5443	3.4230	3.4564	3.4396	3.4068	3.3910
	0.0776	0.0363	0.0741	0.0184	0.0180	0.0710
			$m=12$			
θ	1.7534	2.5882	9.2086	3.6931	2.1241	1.8557
	1.7658	1.3405	13.6102	1.1604	0.4546	1.2639
σ	3.5557	3.4503	3.4776	3.4638	3.4369	3.4238
	0.0518	0.0290	0.0597	0.0147	0.0143	0.0561

TABLE 2. ML estimates of θ and σ for both cases based on only lower record values and upper record values with their inter-record times. The first and second rows represent the average estimates and the estimated risks, respectively.

	Based on only lower records	Based on upper records and inter-record times		Based on only lower records	Based on upper records and inter-record times
	MLE	MLE		MLE	MLE
			$n=1000$		
$\theta=2$	$9.9\ 10^{12}$	1.9122	$\theta=6$	$3.46\ 10^{11}$	8.2471
	$8.27\ 10^{28}$	17.0264		$2.40\ 10^{26}$	369.4632
$\sigma=5$	3.5836	5.1213	$\sigma=3$	2.8148	3.0772
	4.5218	0.5410		0.6871	0.1684
			$n=5000$		
$\theta=2$	3.1924	1.9408	$\theta=6$	8.7099	5.1805
	891.4567	8.1063		341.1306	99.0856
$\sigma=5$	4.4878	5.0835	$\sigma=3$	2.8346	3.0608
	3.3562	0.2466		0.6313	0.0904
			$n=10000$		
$\theta=2$	2.3113	1.7880	$\theta=6$	7.9900	5.7098
	3.0018	2.7874		263.6595	26.2353
$\sigma=5$	4.5651	5.0941	$\sigma=3$	2.8905	3.0376
	3.3246	0.1816		0.5967	0.0592

References

[1] Ahmadi, J. and Doostparast, M. (2006). Bayesian estimation and prediction for some life distributions based on record values. *Statistical Papers*, 47(3), 373-392.

[2] Asgharzadeh, A. and Fallah, A. (2011). Estimation and Prediction for Exponentiated Family of Distributions Based on Records. *Communications in Statistics-Theory and Methods*, 40, 68-83.

[3] Wang, L. and Shi, Y. (2013). Reliability analysis of a class of exponential distribution under record values. *Journal of Computational and Applied Mathematics*, 239, 367-379.

[4] Kızılaslan, F. and Nadar, M. (2013). Estimation with the generalized exponential distribution based on record values and inter-record times. *Journal of Statistical Computation and Simulation*, DOI:10.1080/00949655.2013.856910.

[5] Arnold, B.C., Balakrishnan, N. and Nagaraja, H.N. (1998). *Records*. John Wiley & Sons, New York.

[6] Lehmann, E.L. (1953). The power of rank tests. *Annals of Mathematical Statistics*, 24, 28-43.

[7] Shakil, M. and Ahsanullah, M. (2012). Review on Order Statistics and Record Values from F^{α} Distributions. *Pakistan Journal of Statistics and Operation Research*, 8(1), 101-120.

[8] Hofmann, G. and Nagaraja, H.N. (2003). Fisher information in record data. *Metrika*, 57, 177-193.

[9] Varian, H.R. (1975). A Bayesian approach to real estate assessment, In: Finberg S.E., Zellner A. (eds) *Studies in Bayesian econometrics and statistics in honour of Leonard J.* Savage, North Holland, Amsterdam.

[10] Soland, R.M. (1969). Bayesian analysis of the Weibull process with unknown scale and shape parameters. I*EEE Transactions on Reliability*, 18, 181-184.

[11] Nadar, M. and Kızılaslan, F. (2014). Estimation and prediction of the Burr type XII distribution based on record values and inter-record times. *Journal of Statistical Computation and Simulation*, DOI:10.1080/00949655.2014.970554.