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A TEST OF GOODNESS OF FIT BASED ON GINI INDEX

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Abstract: This paper introduces a general goodness-of-fit test based on the estimated Gini index. The exact and asymptotic distribution of the test statistic are presented. Then goodness-of-fit tests for the normal, exponential, uniform and Laplace distributions are presented. The powers of the proposed tests under various alternatives are compared with the other tests via simulation study. The use of our test is shown in real examples.

Key words: Goodness of fit tests, Gini index, Normal, Exponential, Uniform, Laplace.
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1. Introduction

In engineering and management sciences studies, it is important to test whether the underlying distribution has a particular form. Most statistical methods assume an underlying distribution in the derivation of their results and inferences. Therefore, methods for checking that the underlying distribution has a special form, i.e. goodness of fit tests, is necessary.

Different methods for goodness of fit tests are introduced by researchers in view of goodness of fit tests based on empirical distribution function, empirical characteristic function, entropy and Kullback-Leibler information, maximum correlations, and divergences.

The goodness of fit tests has been discussed by many authors including Saniga and Miles (1979), Dudewicz and Van der Meulen (1981), Read (1984), D'Agostino and Stephens (1986), Arizono and Ohta (1989), Baglivo et al. (1992), Huang (1997), Aerts et al. (1999), Kim (2000), Esteban et al. (2001), Zhang (2002), Fortiana and Grané (2003), Chen et al. (2003), Pouet (2004), Choi et al. (2004), Hunter et al. (2008), Christensen and Sun (2010), Cheng et al. (2010), Alizadeh Noughabi (2010), Ma et al. (2011), and Alizadeh Noughabi and Arghami (2011a,b, 2013a,b,c). Moreover, some tests for censored data are proposed by authors; see, for example, Balakrishnan et al. (2004), Balakrishnan et al. (2007), Habibi Rad et al. (2011), Pakyari and Balakrishnan (2012), Lin et al. (2008), and Pakyari and Balakrishnan (2013).

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The Gini coefficient is one of the indices most widely used to measure of income inequality. It is defined as:

$$G = 1 - 2\int_0^1 L(p)dp,$$

where is the well-known Lorenz function,

$$L(p) = \frac{1}{E(X)}\int_0^p F^{-1}(t)dt.$$

An equivalent expression for the Gini index is used by Giles (2004) as

$$G = \frac{\int_m^M F(y)(1-F(y))dy}{\mu},$$

where the variable is defined on a real interval (m, M) with $0 \leq m < M < \infty$, and μ is the expected value of the variable.

Suppose that an IID sample of size n is drawn randomly from the population, and F_n denotes the empirical distribution function. Let $x_1, x_2, \ldots, x_n$ be a random sample and $x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(n)}$ be the order statistics obtained from the sample, the usual estimator is

$$\hat{G}_n = \frac{\int_m^M y(2F_n(y)-1)dF_n(y)}{\bar{X}} = \sum_{i=1}^n \frac{(2i-n)x_{(i)}}{n\sum_{j=1}^n x_j}.$$

Gail and Gastwirth (1978) introduced a scale-free goodness of fit test for the exponential distribution based on the Gini statistic. They showed that their test has a good power. Next Jammalamadaka and Goria (2004) used spacings and introduced a test of goodness of fit based on Gini index of spacings. In this paper, we introduce a general test of goodness of fit based on the Gini index of data.

In section 2, a general goodness of fit test based on Gini index is introduced. Also properties of proposed test are discussed. Several examples of goodness of fit tests of scale and location-scale families are considered in Sections 3. The power values of the proposed tests compared with the competitor tests by using simulation study. Section 4 contains the use of the proposed test via real examples.

2. Test statistics and its properties

Let $X_1, \ldots, X_n$ be a random sample from an unknown distribution F with a probability density function $f(x)$. Let $F_0(x;\theta)$ be a parametric family of distributions with probability density function $f_0(x;\theta)$. The hypothesis of interest is

$$H_0 : f(x) = f_0(x;\theta), \text{ for some } \theta \in \Theta,$$

and the alternative to H_0 is

$$H_1 : f(x) \neq f_0(x;\theta), \text{ for any } \theta \in \Theta.$$

Without loss of generality, by means of the probability integral transformation $u_i = F_0(x_i), i = 1, 2, \ldots, n$, we can reduce the above problem of goodness of fit, to testing the hypothesis of uniformity on the unit interval, i.e.,

$$H_0 : f(u) = 1, 0 < u < 1 \text{ against the alternative } H_0 : f(u) \neq 1, 0 < u < 1.$$

We use the Gini index as a test statistic for the above problem of goodness of fit test. The usual estimator of Gini index is considered and consequently the proposed test statistic is as

$$\begin{aligned}\hat{G}_n &= \sum_{i=1}^{n} \frac{(2i-n)u_{(i)}}{n\sum_{j=1}^{n} u_j} \\ &= \sum_{i=1}^{n} \frac{(2i-n)F_0(x_{(i)};\hat{\theta})}{n\sum_{j=1}^{n} F_0(x_i;\hat{\theta})},\end{aligned}$$

where $\hat{\theta}$ is a reasonable estimate of θ.

The exact and the asymptotic distributions of $\hat{G}_n$ under the null hypothesis of uniformity are stated in the following theorems.

Theorem 1. *Let $u_1, u_2, \ldots, u_n$ be a random sample from uniform distribution. Then we have*

$$F_{\hat{G}_n}(t) = P(\hat{G}_n \le t) = \int_0^{\tau(a_n)} \int_0^{\tau(a_{n-1})} \cdots \int_0^{\tau(a_1)} e^{-\sum_{i=1}^{n} t_j} dt_1 dt_2 \ldots dt_n,$$

where $a_i = (n+1-i)(i-nt)$ for $1 \le i \le n$ and

$$\tau(a_j) = \begin{cases} \infty & \text{if } a_j \le 0 \\ -\sum_{i=j+1}^{n} a_i t_i / a_j & \text{if } a_j > 0. \end{cases}$$

Proof. *See Martinez-Camblor and Correal (2009) for more details.*

According to Martinez-Camblor and Correal (2009), for very small sample sizes the $F_{\hat{G}_n}$ can be computed easily but the complexity of the problem increases dramatically with sample size (for $n \ge 5$ the problem begins to be embarrassing). Therefore, the exact distribution can not be used in practical problems. The next theorem states that the asymptotic distribution of the test statistic is normal.

Theorem 2. *Let $u_1, u_2, \ldots, u_n$ be a random sample from uniform distribution. Then we have the convergence*

$$\sqrt{n}\frac{\hat{G}_n - 1/3}{\sqrt{8/135}} \xrightarrow{D} N(0,1).$$

Proof. *See Martinez-Camblor and Correal (2009).*

3. Test for some distributions

In this section, we consider normal, exponential, uniform and Laplace distributions and use the proposed test statistic for testing these distributions.

3.1. Competitor tests

Since the proposed test is a general test, it is natural that the competitors also be general tests. The competitor tests are chosen from the class of tests discussed in D'Agostino and Stephens (1986). The test statistics of competitor tests are as follows.

The Kolmogorov–Smirnov, Cramer–von Mises, Kuiper and Anderson-Darling test statistics are respectively:

$$\begin{aligned}KS &= \max_{1\le i\le n}\left\{\frac{i}{n} - Z_i, Z_i - \frac{i-1}{n}\right\}, \\ CH &= \frac{1}{12n} + \sum_{i=1}^{n}\left(\frac{2i-1}{2n} - Z_i\right)^2, \\ V &= \max_{1\le i\le n}\left\{\frac{i}{n} - Z_i\right\} + \max_{1\le i\le n}\left\{Z_i - \frac{i-1}{n}\right\}, \\ AD &= -n - \frac{\sum_{i=1}^{n}(2i-1)\left\{\ln(Z_i) + \ln(1 - Z_{n-i+1})\right\}}{n},\end{aligned}$$

where $Z_i = F_0(x_{(i)}; \hat{\theta}), i = 1, \ldots, n$ and F_0 is the cdf under the null distribution.

3.2. Testing normality

We have the following test statistic for testing normality.

$$\hat{G}_n = \sum_{i=1}^{n} \frac{(2i-n)\, F_0(x_{(i)}; \hat{\theta})}{n \sum_{i=1}^{n} F_0(x_i; \hat{\theta})}$$

where F_0 is normal distribution function and $\hat{\theta} = (\hat{\mu}, \hat{\sigma})$ where

$$\hat{\mu} = \bar{X} = \tfrac{1}{n} \textstyle\sum_{i=1}^{n} X_i \qquad \hat{\sigma} = s = \sqrt{\tfrac{1}{n} \textstyle\sum_{i=1}^{n} (X_i - \bar{X})^2}$$

It is obvious that the test statistic is invariant with respect to location and scale transformations.

Monte Carlo methods were used to obtain the critical values of our procedure. Table 1 gives the critical values of the proposed statistic for testing normality.

TABLE 1. Critical values of the proposed statistic for testing normality.

	Significance level							
n	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
5	0.4317	0.4471	0.4602	0.4753	0.5319	0.5339	0.5351	0.5359
10	0.3499	0.3643	0.3770	0.3903	0.4451	0.4481	0.4500	0.4517
15	0.3322	0.3440	0.3544	0.3649	0.4135	0.4170	0.4195	0.4218
20	0.3256	0.3357	0.3441	0.3532	0.3972	0.4007	0.4032	0.4057
25	0.3221	0.3319	0.3390	0.3469	0.3870	0.3905	0.3930	0.3954
30	0.3204	0.3288	0.3355	0.3426	0.3799	0.3832	0.3857	0.3883
40	0.3199	0.3260	0.3316	0.3379	0.3706	0.3738	0.3761	0.3788
50	0.3190	0.3248	0.3298	0.3352	0.3648	0.3679	0.3701	0.3728

TABLE 2. Power comparisons of normal goodness of fit tests with size 0.05. $\widehat{G}_n^1$ denotes One-sided and $\widehat{G}_n^2$ denotes Two-sided

	n=10						n=20					
Alternatives	KS	CH	V	AD	$\widehat{G}_n^1$	$\widehat{G}_n^2$	KS	CH	V	AD	$\widehat{G}_n^1$	$\widehat{G}_n^2$
Normal	0.051	0.051	0.054	0.047	0.050	0.050	0.049	0.051	0.048	0.051	0.050	0.050
Laplace	0.142	0.158	0.142	0.159	0.181	0.127	0.326	0.425	0.353	0.467	0.336	0.243
Logistic	0.073	0.080	0.071	0.083	0.090	0.069	0.087	0.106	0.090	0.113	0.142	0.098
Cauchy	0.580	0.618	0.589	0.618	0.605	0.538	0.847	0.880	0.865	0.882	0.889	0.853
t_2	0.273	0.304	0.274	0.310	0.318	0.256	0.457	0.518	0.486	0.535	0.574	0.495
t_3	0.164	0.182	0.163	0.190	0.207	0.160	0.260	0.309	0.277	0.327	0.382	0.307
t_5	0.100	0.112	0.099	0.116	0.125	0.093	0.131	0.161	0.141	0.174	0.219	0.159
Uniform	0.066	0.074	0.081	0.080	0.157	0.097	0.100	0.144	0.150	0.171	0.349	0.249
Beta(2,2)	0.046	0.044	0.048	0.046	0.091	0.057	0.053	0.058	0.064	0.058	0.160	0.096

The powers of the normality tests based on $\hat{G}_n$, KS, V, CH and AD statistics for some alternatives and samples of size $n = 10, 20$ are estimated and reported in Table 2.

We observe that the proposed test performs very well compared with the other tests.

3.3. Testing exponentiality

We have the following test statistic for testing exponentiality.

$$\hat{G}_n = \sum_{i=1}^{n} \frac{(2i-n)\, F_0(x_{(i)};\hat{\theta})}{n\sum_{i=1}^{n} F_0(x_i;\hat{\theta})},$$

where F_0 is the exponential distribution function and $\hat{\theta} = \frac{1}{n}\sum_{i=1}^{n} X_i$.

The test statistic is invariant with respect to transformations of scale. By Monte Carlo methods the critical points and power values of our test are obtained and reported in Tables 3 and 4, respectively.

Table 3. Critical values of the proposed statistic for testing exponentiality.

	Significance level							
n	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
5	0.2848	0.3079	0.3309	0.3588	0.6034	0.6394	0.6714	0.7127
10	0.2578	0.2795	0.2982	0.3217	0.4948	0.5202	0.5433	0.5689
15	0.2587	0.2760	0.2926	0.3115	0.4546	0.4758	0.4937	0.5152
20	0.2633	0.2782	0.2928	0.3094	0.4328	0.4504	0.4662	0.4850
25	0.2671	0.2807	0.2936	0.3081	0.4186	0.4344	0.4487	0.4650
30	0.2690	0.2827	0.2939	0.3078	0.4090	0.4232	0.4359	0.4503
40	0.2738	0.2856	0.2959	0.3079	0.3959	0.4089	0.4200	0.4330
50	0.2772	0.2887	0.2982	0.3091	0.3875	0.3992	0.4083	0.4198

Table 4. Power comparisons of exponential goodness of fit tests with size 0.05. $\widehat{G}_n^1$ denotes One-sided and $\widehat{G}_n^2$ denotes Two-sided

	n=10						n=20					
Alternatives	KS	CH	V	AD	$\widehat{G}_n^1$	$\widehat{G}_n^2$	KS	CH	V	AD	G1	G2
Exponential	0.051	0.051	0.054	0.047	0.050	0.050	0.051	0.048	0.049	0.052	0.050	0.050
Gamma(2)	0.211	0.240	0.200	0.176	0.376	0.242	0.406	0.486	0.372	0.441	0.673	0.523
Gamma(3)	0.457	0.536	0.445	0.448	0.715	0.551	0.811	0.891	0.780	0.876	0.967	0.921
Weibull(2)	0.500	0.609	0.515	0.518	0.742	0.593	0.848	0.930	0.850	0.915	0.971	0.932
Weibull(3)	0.899	0.968	0.931	0.947	0.988	0.963	0.999	1.000	1.000	1.000	1.000	1.000
Normal (5,1)	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Uniform	0.280	0.362	0.356	0.289	0.384	0.272	0.528	0.673	0.668	0.619	0.622	0.497
Beta(2,1)	0.837	0.947	0.934	0.921	0.956	0.904	0.995	0.999	0.999	0.999	1.000	0.999
Beta(2,2)	0.622	0.764	0.699	0.691	0.839	0.720	0.933	0.987	0.971	0.983	0.992	0.978
Log-normal	0.097	0.101	0.088	0.079	0.097	0.065	0.138	0.152	0.143	0.134	0.129	0.083
$\chi^2_{(1)}$	0.260	0.290	0.217	0.475	0.503	0.405	0.471	0.524	0.391	0.701	0.748	0.656

3.4. Testing uniformity

We have the following test statistic for testing uniformity.

$$\hat{G}_n = \sum_{i=1}^{n} \frac{(2i-n)\, u_{(i)}}{n \sum_{i=1}^{n} u_i}$$

The critical values of our test are given in Table 5.

TABLE 5. Critical values of the proposed statistic for testing uniformity.

	Significance level							
n	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
5	0.2742	0.2970	0.3205	0.3516	0.6206	0.6607	0.6976	0.7362
10	0.2474	0.2699	0.2898	0.3146	0.5084	0.5379	0.5643	0.5920
15	0.2483	0.2680	0.2855	0.3063	0.4651	0.4888	0.5087	0.5334
20	0.2516	0.2690	0.2848	0.3030	0.4418	0.4615	0.4782	0.4990
25	0.2563	0.2713	0.2858	0.3026	0.4270	0.4450	0.4597	0.4770
30	0.2591	0.2743	0.2878	0.3033	0.4160	0.4325	0.4472	0.4641
40	0.2648	0.2787	0.2900	0.3037	0.4020	0.4165	0.4292	0.4433
50	0.2717	0.2833	0.2933	0.3055	0.3932	0.4061	0.4173	0.4308

TABLE 6. Power comparisons of uniform goodness of fit tests with size 0.05.

	n=10						n=20					
Alternatives	KS	CH	V	AD	$\widehat{G}_n^1$	$\widehat{G}_n^2$	KS	CH	V	AD	$\widehat{G}_n^1$	$\widehat{G}_n^2$
Uniform	0.051	0.051	0.052	0.048	0.050	0.050	0.048	0.049	0.049	0.053	0.050	0.050
Beta(2,1)	0.390	0.440	0.245	0.421	0.549	0.400	0.686	0.759	0.474	0.752	0.864	0.762
Beta(3,1)	0.806	0.866	0.569	0.857	0.908	0.815	0.988	0.996	0.934	0.996	0.999	0.995
Beta(3,2)	0.204	0.180	0.349	0.122	0.634	0.462	0.499	0.486	0.682	0.443	0.943	0.871
Beta(3,.5)	0.990	0.997	0.917	0.998	0.990	0.972	1.000	0.999	1.000	1.000	1.000	1.000
Beta(2,.5)	0.894	0.938	0.666	0.960	0.874	0.782	0.995	0.999	0.959	0.999	0.995	0.989
Beta(2,2)	0.041	0.026	0.182	0.014	0.201	0.110	0.065	0.047	0.358	0.039	0.394	0.252
Beta(3,3)	0.046	0.026	0.410	0.012	0.442	0.281	0.148	0.136	0.769	0.151	0.793	0.643

The estimated powers of the uniformity tests based on $\hat{G}_n$, KS, V, CH and AD statistics for samples of size $n = 10, 20$ are reported in Table 6.

We see that the proposed test performs very well compared with the other tests. The difference of powers the proposed test and other tests are substantial.

3.5. Test for Laplace distribution

The hypothesis of interest is
$H_0 : f(x) = f_0(x;\alpha,\beta) = \frac{1}{2\beta^2} \exp\left\{-\frac{|x-\alpha|}{\beta}\right\}, -\infty < x < \infty$ for some $(\alpha,\beta) \in \Theta = \mathbb{R} \times \mathbb{R}^+$, where α and β are unknown. The alternative to H_0 is

$$H_1 : f(x) \neq f_0(x;\alpha,\beta), \text{ for any } (\alpha,\beta) \in \Theta.$$

The test statistics is:

$$\hat{G}_n = \sum_{i=1}^{n} \frac{(2i-n)\, F_0(x_{(i)};\hat{\theta})}{n \sum_{i=1}^{n} F_0(x_i;\hat{\theta})},$$

where F_0 is Laplace distribution function and $\hat{\theta} = (\hat{\alpha}, \hat{\beta})$ where

$$\hat{a} = median(x_1, x_2, \ldots, x_n) \;\; ; \;\; \hat{\beta} = \frac{1}{n}\sum_{i=1}^{n} |x_i - \hat{\alpha}|.$$

It is obvious that the test statistic is invariant with respect to location and scale transformations.

The critical values of our procedure are obtained by Monte Carlo methods. Table 7 gives the critical values of the proposed statistic.

Table 7. Critical values of the proposed statistic for test of Laplace distribution.

	Significance level							
n	0.01	0.025	0.05	0.10	0.90	0.95	0.975	0.99
5	0.3930	0.4132	0.4329	0.4573	0.5758	0.5824	0.5869	0.5908
10	0.3446	0.3605	0.3730	0.3872	0.4620	0.4694	0.4750	0.4805
15	0.3250	0.3364	0.3470	0.3586	0.4309	0.4391	0.4456	0.4522
20	0.3224	0.3315	0.3403	0.3500	0.4085	0.4152	0.4203	0.4264
25	0.3156	0.3255	0.3332	0.3423	0.3987	0.4054	0.4106	0.4169
30	0.3161	0.3246	0.3315	0.3393	0.3885	0.3941	0.3992	0.4046
40	0.3149	0.3217	0.3279	0.3347	0.3780	0.3832	0.3874	0.3923
50	0.3149	0.3211	0.3265	0.3323	0.3713	0.3762	0.3802	0.3850

The powers of the normality tests based on $\hat{G}_n$, KS, V, CH and AD statistics for some alternatives and samples of size $n = 10, 20$ are estimated and reported in Table 8.

We observe that the proposed tests perform very well compared with the other tests for some alternatives.

Table 8. Power comparisons of Laplace goodness of fit tests with size 0.05.

	n=10						n=20					
Alternatives	KS	CH	V	AD	$\widehat{G}_n^1$	$\widehat{G}_n^2$	KS	CH	V	AD	$\widehat{G}_n^1$	$\widehat{G}_n^2$
Normal	0.051	0.051	0.054	0.047	0.086	0.058	0.089	0.084	0.103	0.073	0.125	0.081
Gamma(2)	0.104	0.121	0.093	0.131	0.159	0.118	0.183	0.212	0.193	0.267	0.239	0.182
Gamma(3)	0.076	0.089	0.076	0.094	0.114	0.091	0.133	0.153	0.142	0.185	0.159	0.125
Weibull(2)	0.063	0.069	0.068	0.067	0.068	0.069	0.116	0.118	0.146	0.120	0.078	0.082
Weibull(3)	0.051	0.057	0.059	0.052	0.084	0.059	0.102	0.090	0.125	0.081	0.087	0.130
Exponential	0.241	0.245	0.202	0.269	0.328	0.250	0.473	0.437	0.461	0.535	0.501	0.411
Uniform	0.099	0.116	0.138	0.106	0.127	0.093	0.244	0.256	0.364	0.253	0.212	0.155
Beta(2,1)	0.105	0.131	0.114	0.128	0.143	0.080	0.216	0.242	0.290	0.267	0.279	0.176
Beta(2,2)	0.065	0.071	0.082	0.064	0.110	0.074	0.154	0.137	0.200	0.129	0.176	0.119
Logistic	0.046	0.050	0.046	0.046	0.070	0.050	0.064	0.056	0.066	0.052	0.094	0.064
Lognormal(0..5)	0.092	0.106	0.091	0.117	0.167	0.122	0.163	0.188	0.158	0.239	0.245	0.182
Lognormal(0.1)	0.354	0.352	0.314	0.401	0.491	0.401	0.681	0.623	0.642	0.733	0.726	0.653
Lognormal(0.2)	0.845	0.818	0.820	0.843	0.868	0.826	0.993	0.981	0.992	0.990	0.987	0.980
t_1	0.325	0.336	0.371	0.357	0.364	0.310	0.516	0.544	0.607	0.565	0.598	0.538
t_3	0.055	0.053	0.064	0.056	0.062	0.061	0.071	0.068	0.079	0.075	0.082	0.077
$\chi^2_{(1)}$	0.565	0.521	0.516	0.553	0.608	0.523	0.900	0.820	0.907	0.886	0.835	0.781

4. Real examples

In this section, we present two real examples to show the behavior of the proposed test in real cases.

Example 1. The following dataset is considered by Bain and Engelhardt (1973), consisting of 33 difference in flood levels between stations on a river.

1.96, 1.97, 3.60, 3.80, 4.79, 5.66, 5.76, 5.78, 6.27, 6.30,6.76, 7.65, 7.84, 7.99, 8.51, 9.18, 10.13, 10.24, 10.25, 10.43, 11.45, 11.48, 11.75, 11.81, 12.34, 12.78, 13.06, 13.29, 13.98, 14.18, 14.40, 16.22, 17.06.

They suggested that the Laplace distribution might provide a good fit. Puig and Stephens (2000) used the EDF tests for fitting a Laplace distribution for the data. They obtained $AD = 0.965$, $CH = 0.155$, $\sqrt{n}KS = 0.917$, $V = 1.241$ and concluded that KS and CH just reject the Laplace assumption for the data at 0.05 level.

For this example, we find $\hat{\alpha} = 10.13$, $\hat{\beta} = 3.361$ and $\hat{G}_n = 0.4088$ and the critical values are 0.3139, 0.3222, 0.3292, 0.3921, 0.3970, and 0.4030 at levels 0.01, 0.025, 0.05, 0.95, 0.975 and 0.99, respectively. Therefore the Laplace assumption is rejected and our procedure confirms the result obtained by KS and CH tests.

Example 2. In this example one real-life data analysis from Lawless (1982) is considered. The following dataset consist failure times for 36 appliances subjected to an automatic life test.

11, 35, 49, 170, 329, 381, 708, 958, 1062, 1167, 1594, 1925, 1990, 2223, 2327, 2400, 2451, 2471, 2551, 2565, 2568, 2694, 2702, 2761, 2831, 3034, 3059, 3112, 3214, 3478, 3504, 4329, 6367, 6976, 7846, 13403.

Ebrahimi et al. (1992) applied the exponential distribution to this data which was satisfactory. Recently the same conclusion has been drawn by Baratpour and Habibi Rad (2012).

For this example, we obtained $\hat{G}_n = 0.3513$ and the critical values are 0.2735, 0.2851, 0.2957, 0.4138, 0.4254, and 0.4385 at levels 0.01, 0.025, 0.05, 0.95, 0.975 and 0.99, respectively. Then the exponential assumption is accepted and the results obtained by previous authors are confirmed.

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