PROBABILITY MODEL FOR MIGRATION SYSTEM

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Abstract

In this paper we have proposed probability model for out-migration. Estimates of parameters involved in the model along with their asymptotic variance are given. Tested suitability of model through observed data.

Key words: Probability Model, Asymptotic Variance, Household, Risk of Migration, Migration in Clusters.

1. Introduction

Migration has become major policy issues in developing countries of Asian and Pacific region. It is fact migration is closely associated with the economic fluctuations, nature of physical environment, social organization of the groups. Migration at micro-level has played a decisive role in the development of theories of migration and investigation of factors affecting movement process, is the best carried out with models. In micro-level studies migration has been studied at community level, village level, household level, or individual level depending upon the objective and availability of data. The most common unit of human grouping is households. The characteristics of households are bound to play an active role in the decision of an individual to move or not to move from a household. Singh, S.N. et al. (1982), Ojha and Pandey (1991) have proposed various probability model to study the trends in rural out-migration at household level.

The aim of the present paper is to develop probability model for the number of migrant aged 15 years and above. Estimates of parameters determined with asymptotic expression for their variance.
2. Model

Let $X$ denote the random number of rural out-migration from a household. Probability model for describing the variation in the number of single male migrants aged 15 years and above has been obtained on the basis of the following assumptions:

(i) $\alpha$ be the probability that a household is exposed to the risk of migration at the survey point and $(1 - \alpha)$ be the probability that a household is not exposed to the risk of migration.

(ii) The $K$ males migrating from a household is follows a decapitated Poisson distribution.

From assumption (i) and (ii), through Johnson and Kotz (1969), the probability model become:

$$P[X = 0] = 1 - \alpha$$

$$P[X = k] = \alpha \left( \frac{\theta^K (e^\theta - 1)^{-1}}{K!} \right), \quad K = 1, 2, ..., \theta > 0.$$  \hspace{1cm} (2.1)

and with probability generating function

$$P_X(s) = 1 - \alpha + \alpha(e^{\theta s} - 1)(e^\theta - 1)^{-1}$$ \hspace{1cm} (2.2)

3. Estimates of Parameters and Their S.E.

Proposed model involves two parameters $\alpha$ and $\theta$. Suppose $Z$ households are observed at random and $Z_n$ denotes the number of households with $n$ migrants and $Z_0$ denote the number of households in $Z$ with no out-migrants. So,

$$\bar{X} = \sum \frac{nZ_n}{Z} \quad \text{and} \quad E(\bar{X}) = \alpha \theta e^\theta (e^\theta - 1)^{-1}$$

we have

$$E(\bar{X}) = E\left( \sum \frac{nZ_n}{Z} \right) = \alpha \theta e^\theta (e^\theta - 1)^{-1}$$ \hspace{1cm} (3.1)

where $\bar{X}$ is the sample mean that is number of out migrants per household. From model (2.1), by observed proportion of household with $K = 0$, we have as estimate of $\alpha:

$$\hat{\alpha} = \frac{Z - Z_0}{Z} = Y \quad \text{(say)}$$ \hspace{1cm} (3.2)
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Then from (3.1) an estimator of $\theta$ can be found by expression, which gives estimate of

$$\hat{\theta} e^\theta (e^\theta - 1)^{-1} = \frac{\bar{X}}{\bar{Z}}$$

(3.3)

in some interactions.

Using the central limit theorem for large $Z$.

$$\sqrt{Z}(\bar{X} - \mu) \approx N(0, V_1^2),$$

where $\mu$ stands for $\alpha \theta e^\theta (e^\theta - 1)^{-1}$ and from (2.2)

$$Var(X) = V_1^2 = P_X'(1) + P_X'(1) - [P_X'(1)^2]$$

$$= \alpha \theta e^\theta (e^\theta - 1)^{-1} [1 + \theta - \alpha \theta e^\theta (e^\theta - 1)^{-1}]$$

is the population variance. Also $\sqrt{Z}(Y - E(Y)) \approx N(0, V_2^2)$ with $V_2^2 = \alpha(1 - \alpha)$.

The differential expression of (3.2) and (3.3) are,

$$d\bar{\alpha} = dY$$

(3.4)

and

$$d\bar{X} = \theta e^\theta (e^\theta - 1)^{-1} d\bar{\alpha} - \alpha e^\theta \left[ \frac{\theta e^\theta - (e^\theta - 1)(\theta + 1)}{(e^\theta - 1)^2} \right] d\hat{\theta}$$

(3.5)

Solving (3.4) and (3.5) in forms of $d\bar{\alpha}$ and $d\hat{\theta}$, squaring and taking expectations, it is found that for large $Z$,

$$\sqrt{Z}(\bar{\alpha} - \alpha) \approx N(0, V_3^2)$$

and

$$\sqrt{Z}(\hat{\theta} - \theta) \approx N(0, V_4^2)$$

where

$$J^2 V_3^2 = \alpha^2 e^{2\theta} \left[ \frac{\theta e^\theta - (e^\theta - 1)(\theta + 1)}{(e^\theta - 1)^4} \right] V_2^2$$

(3.6)

and

$$J^2 V_4^2 = V_1^2 + \left[ \theta e^\theta (e^\theta - 1)^{-1} \right]^2 V_2^2 - 2\theta e^\theta (e^\theta - 1)^{-1} V_1 V_2$$

(3.7)

where $J$ being the Jacobian of the transformation and given by following expression

$$J = \frac{\alpha e^\theta \left[ \theta e^\theta - (e^\theta - 1)(1 + \theta) \right]}{(e^\theta - 1)^2}$$

(3.8)

From (3.6) and (3.8), trivially $V_3^2 = V_2^2 = \alpha(1 - \alpha)$.

It also may be noted that the covariance between $\sqrt{Z}(\bar{\alpha} - \alpha)$ and $\sqrt{Z}(\hat{\theta} - \theta)$ is zero.

Sukhatme et. al. (1976) have pointed out that the sample mean and sample proportion are the consistent and unbiased estimates of population mean and population proportion respectively. Therefore, $\bar{\alpha}$ and $\hat{\theta}$ are the consistent and unbiased.
4. Conclusions

The proposed probability model has been applied to the migration data taken from the survey "Rural Development and Population Growth (R.D.P.G.)" which was conducted by Population Research Centre Varanasi (India) for Remote and Growth Centre type villages.

From table-I, the observed value of $\chi^2$ as found to be insignificant for the Remote type villages at 5 percent level of significances whereas for Growth Centre type villages the value of $\chi^2$ also insignificant at 1 percent level of significance. Thus it may conclude that propose model describe satisfactorily well to the out-migrants.

It will be observed that the value of risk of migration that is $\hat{\alpha}$ has more variation for the both types of villages but the value of $\hat{\beta}$ are nearly same for the both types of villages. It may be conclude that most of the variation in the migration rates is mainly ude to value of $\hat{\alpha}$ rather than the value of $\hat{\beta}$.

References


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